$$y_A - y_L = v_o' t_{L,1}' - \frac{1}{2} g (t_{L,1}')^2 .$$

The first equation gives

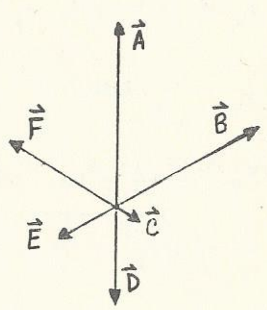
$$t_{H,1}' = \frac{v_o' \pm \sqrt{(v_o')^2 - 2g(y_A - y_H)}}{g}$$

while the second equation gives an analogous equation for $t_{L,1}'$. In each case, the lower (-) sign must be chosen, since the tomatoes are still traveling upward. When numerical values are inserted, we find $t_{H,1}' = 0.55$ s and $t_{L,1}' = 1.00$ s. Therefore Alice must be out of the way within 0.55 s after the tomatoes are thrown.

(f) To find the times $t_{H,2}'$ and $t_{L,2}'$ when the tomatoes strike Alice, we need only use the upper (+) sign in the displayed equation for $t_{H,1}'$ in part (e) and in the analogous equation for $t_{L,1}'$. We find $t_{H,2}' = 5.57$ s and $t_{L,2}' = 5.13$ s. That is, Lou's tomato strikes Alice first, 5.13 s after it was thrown (and 4.13 s after missing on the way up). Its y-velocity is given by $v_o' - g t_{L,2}' = -20.3$ m/s, so it hits her with a speed of 20.3 m/s. Hugh's tomato strikes her 0.44 s later (5.57 s after it was thrown and 5.02 s after missing on the way up), with a speed of $|v_o' - g t_{H,2}'| = 24.6$ m/s.

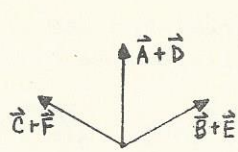
CHAPTER THREE

3-8



In the upper figure, we have labeled the vectors $\vec{A}$ through $\vec{F}$, proceeding clockwise from $\vec{A}$. As indicated in the text, the magnitudes A-F are 6, 5, 1, 3, 2, and 4, respectively. The resultant vector

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E} + \vec{F}$$
$$= (\vec{A} + \vec{D}) + (\vec{B} + \vec{E}) + (\vec{C} + \vec{F}),$$



as shown in the lower figure. Each partial sum has magnitude 3. It may be seen from the lower figure that $(\vec{B} + \vec{E}) + (\vec{C} + \vec{F}) = (\vec{A} + \vec{D})$. Therefore $R = 2(\vec{A} + \vec{D})$, which is a vector with the same magnitude (6) and direction (vertically upward) as $\vec{A}$. That is, $\vec{R} = \vec{A}$.

3-17

$\vec{v} = v\hat{v}$ so

$$\vec{a} = \frac{d\vec{v}}{dt} = \hat{v}\frac{dv}{dt} + v\frac{d\hat{v}}{dt} = \hat{v}\frac{dv}{dt} + v^2\left(\frac{d\hat{v}}{ds}\right)$$

For v = constant, $a = v^2\left|\frac{d\hat{v}}{ds}\right| = \frac{v^2}{R}$ where R is the radius of curvature. The radius of curvature is smallest at D, so the acceleration is largest there.

3-22

(a) The initial velocity components are $v_{xo} = v$ and $v_{yo} = 0$, so $x(t) = vt$ and $y(t) = gt^2/2$. At t_f, x = L and y = d, so $t_f = L/v$ and

$$d = \frac{1}{2}g(L/v)^2$$

Solving for v, we find

$$v = L\sqrt{g/2d}$$

(b) $g = 9.8\ m/s^2$, $L = 3 \times 10^2$ m and d = 1.30 m. Then v = 582 m/s

(c) d = 1.70 m, v = 582 m/s. $L = v\sqrt{2d/g}$ = 343 m

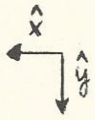
3-23

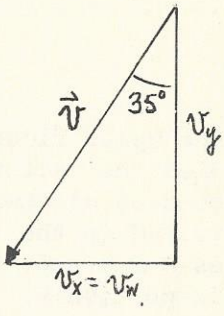
$v_w = 4.5$ m/s, $v_x = v_w$

$v_y = v_w/\tan(35^o)$

v_y = 6.43 m/s = 14.3 mi/hr

$v = \sqrt{v_x^2 + v_y^2}$ = 7.84 m/s = 17.4 mi/hr



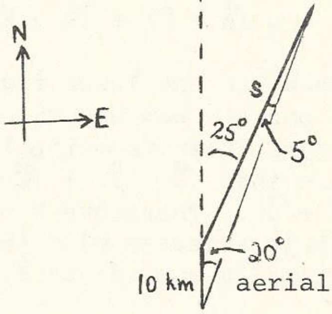


3-24

(a) Applying the law of sines to the aerial view, we have

$$\frac{s}{\sin(20^o)} = \frac{10\ km}{\sin(5^o)}$$

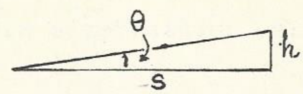
where s is the unknown distance. This gives s = 39.2 km.



aerial view

(b) Referring to the side view,

$$\tan\theta = \frac{h}{s}$$



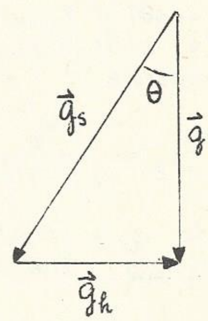
side view

With $\theta = 8^\circ$, we find $h = s \tan(8^\circ)$ $= 39.2\text{ km} \tan(8^\circ) = \underline{5.51\text{ km}}$.

(NOTE: The solution presented here assumes a flat earth. If the earth's curvature is taken into account, the solution requires the use of spherical trigonometry. The same result (to three significant figures) is obtained for the distance s; the computed height is increased by 0.12 km: the height obtained when the earth's curvature is neglected is about 2% low.)

3-25

Let $\vec{g}_s$ and $\vec{g}_h$ be the constituent vectors along the string, and in the horizontal plane, respectively, as shown in the figure. Then $\vec{g} = \vec{g}_s + \vec{g}_h$, where $g_s = g \sec\theta$ directed down along the string, and $g_h = g \tan\theta$ directed toward axis of motion. Since $g_h = a = v^2/r$, we find



$$g_h/g = \underline{\tan\theta} = \underline{v^2/rg}$$

3-26

(a) Drop $= d = \frac{1}{2}gt_f^2$, so $t_f = \sqrt{2d/g}$. Horizontal distance $= L = v_o t_f$, so

$$v_o = L/t_f = L\sqrt{g/2d}$$

With L = 14 m, d = 6 m, we find $v_o = \underline{12.7\text{ m/s}}$.

(b) The impact angle is given by

$$\tan^{-1}(v_{fy}/v_x) = \tan^{-1}(gt_f/v_o) = \tan^{-1}(\sqrt{2gd}/v_o).$$

With the given angle of 35° and d = 6 m, we find $v_o = \sqrt{2gd}/\tan(35^\circ)$ $= \underline{15.5\text{ m/s}}$.

(c) The two estimates of v_o differ by 2.8 m/s or 20% of their average. Their average is 36% smaller than 22 m/s.

(d) Gravity would appear abnormally large. By examining the equations in parts (a) and (b), we can see that a larger effective value of g would bring the two trajectory-based speed estimates into closer agreement with the direct estimate.

3-27

(a) The outfielder's maximum range $R_{max} = v_o^2/g$, so the flight time ${}_1 = R_{max}/[v_o \cos(45^o)] = \underline{\sqrt{2}v_o/g}$.

(b) For a throw to the relay man,

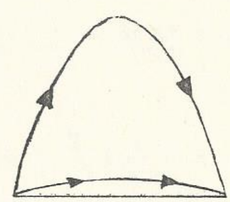
$$R = \frac{v_o^2 \sin 2\theta}{g} = \frac{1}{2}R_{max} = \frac{v_o^2}{2g}$$

Therefore, $\sin 2\theta = \frac{1}{2}$, which gives $\theta = 15^o$ or 60^o. Evidently the choice $\theta = 15^o$ gives the shorter flight time. Each flight (the outfielder's throw and the relay throw) requires a time $R/(v_o \cos\theta) = R_{max}/[2v_o \cos(15^o)]$, so the overall flight time is $R_{max}/[v_o \cos(15^o)] = v_o/[g \cos(15^o)]$. The total elapsed time t_2 is therefore

$$t_2 = \frac{v_o}{g \cos(15^o)} + \Delta t = \frac{1.04 v_o}{g} + \Delta t$$

(c) For $v_o = 35$ m/s and $t = 0.5$ s, we find $t_1 = \underline{5.05 \text{ s}}$ and $t_2 = \underline{4.20 \text{ s}}$. The relay method is quicker.

3-28



(a) The range is given by $R(\theta) = (v_o^2 \sin 2\theta)/g$, with $R = 40$ m and $v_o = 30$ m/s, we find $\sin 2\theta = gR/v_o^2 = 0.4356$. Then $\theta_1 = \underline{77.1^o}$, $\theta_2 = \underline{12.9^o}$.

(b) The time of flight is given by $t = R/(v_o \cos\theta)$. When $\theta = \theta_1$, $t = t_1 = 5.97$ s. When $\theta = \theta_2$, $t = t_2 = 1.37$ s. Hugh should throw the high ball first, wait a time $t_1 - t_2 = \underline{4.60 \text{ s}}$, and then throw the low ball. They will land $\underline{1.37 \text{ s}}$ after the second ball is thrown.

3-29

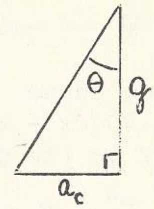
(a) Using $R_e = 6.37 \times 10^6$ m and $T = 86200$ s, we find $a = v^2/R_e = 4\pi^2 R_e/T^2 = 3.38 \times 10^{-2}$ m/s^2. Therefore $a/g = 3.44 \times 10^{-3}$. Since $g_{eff} = g - a$, the fractional dimunition is $(g - g_{eff})/g = a/g = \underline{0.344\%}$. (Note: $g = 9.846$ m/s^2, $g_{eff} = 9.80$ m/s^2)

(b) In order that $g_{eff} = 0$, we need $a = g = 4\pi^2 R_e/T_1^2$. Solving for T_1, we find $T_1 = 2\pi\sqrt{R_e/g} = 5.05 \times 10^3$ s $= \underline{84.2 \text{ min}}$.

(c) Since an orbiting satellite is weightless, its period equals T_1.

3-30

(a) Because the net acceleration must be horizontal, we have $\tan\theta = a_c/g = v^2/rg$. Therefore $\theta = \tan^{-1}(v^2/rg)$.



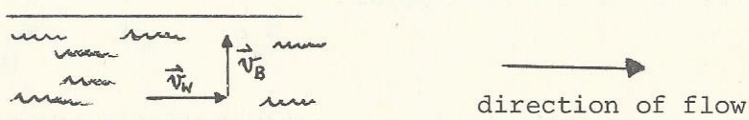
(b) With $v = 60$ m/s and $r = 1.0 \times 10^3$ m, we find

$$\theta = \tan^{-1}\left(\frac{3600}{10^3 \times 9.8}\right) = \underline{20.2^\circ}$$

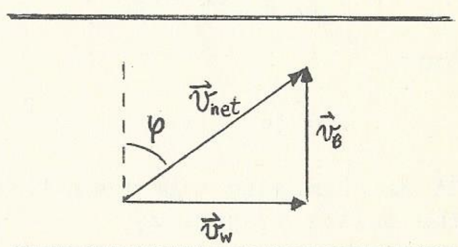
(c) This last question will challenge even the best students at this stage in their study of physics. A full explanation involves recognizing that the magnitude of the lift force must be increased, and then invoking the direct relationship between lift and drag. To balance a larger drag force at the same speed requires a greater applied power.

3-31

(a)



(b) and (c) The velocity with respect to the shore is given by $\vec{v}_{net} = \vec{v}_B + \vec{v}_W$. Since $\vec{v}_B$ and $\vec{v}_W$ are perpendicular, we have $v_{net} = \sqrt{v_B^2 + v_W^2} = \sqrt{3^2 + 4^2} = \underline{5 \text{ m/s}}$. The angle φ shown in the figure is determined by $\tan\varphi = v_W/v_B$. For the speeds given, we find $\varphi = \underline{53.1^\circ}$. The boat moves along a line directed 53.1° downstream from "straight across".



(d) Letting D = distance downstream, we have $D/100 \text{ m} = v_W/v_B = 4/3$, so that $D = \underline{133 \text{ m}}$.

3-32

We use coordinate axes with $\hat{x}$ east, $\hat{y}$ north and $\hat{z}$ vertically upward. We let $\vec{v}_T$ be the velocity of the truck with respect to the ground and $\vec{v}_P$ be the velocity of the police car with respect to the ground. According to the information given, we have

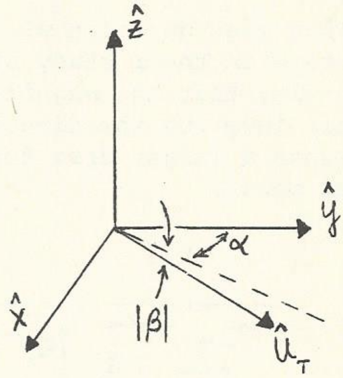
$$\vec{v}_T = 0\hat{x} + (90 \text{ km/hr}) \cos(5.7^\circ)\hat{y} - (90 \text{ km/hr}) \sin(5.7^\circ)\hat{z} \quad \text{and}$$

$$\vec{v}_P = (-80 \text{ km/hr}) \sin(30^\circ)\hat{x} - (80 \text{ km/hr}) \cos(30^\circ)\hat{y} + 0\hat{z}$$

Reducing these, we find $\vec{v}_T = (89.6\hat{y} - 8.94\hat{z})$ km/hr and $\vec{v}_P = (-40.0\hat{x} - 69.3\hat{y})$ km/hr. The velocity $\vec{u}_T$ of the truck with respect to the police car is given by

$$\vec{u}_T = \vec{v}_T - \vec{v}_P = (40.0\hat{x} + 158.8\hat{y} - 8.9\hat{z}) \text{ km/hr}$$

To express this in magnitude-and-direction form, we use $u_T = \sqrt{u_{Tx}^2 + u_{Ty}^2 + u_{Tz}^2}$ and we find u_T = 164 km/hr. Referring to the figure at the right, we have $\tan\alpha = u_{Tx}/u_{Ty} = 0.252$. Therefore $\alpha = \underline{14.1^\circ}$. The elevation angle β is determined by $\sin\beta = u_{Tz}/u_T = -0.0543$, which gives $\beta = -3.11^\circ$. With respect to the police car, the truck's speed is 164 km/hr; its compass heading is 14.1° east of north, and its dip angle is 3.1° below horizontal.



3-33

(a) The position of the ball as a function of time is given by

$$x = (v_o \cos\theta)t$$

and

$$y = (v_o \sin\theta)t - \frac{1}{2}gt^2$$

If we eliminate t between these equations, we get the equation of the ball's trajectory

$$y = x \tan\theta - gx^2/(2v_o^2 \cos^2\theta)$$

The ball lands in the stands when $y = x \tan\alpha$, so we must solve the equation

$$x \tan\alpha = x \tan\theta - gx^2/(2v_o^2 \cos^2\theta)$$

The solutions are $x = 0$ and $x = R_x$, where

$$R_x = \frac{2v_o^2}{g}\cos^2\theta(\tan\theta - \tan\alpha) = \frac{2v_o^2}{g}(\sin\theta\cos\theta - \tan\alpha\cos^2\theta)$$

(b) R_x is maximized when $dR_x/d\theta = 0$.

$$\frac{dR_x}{d\theta} = \frac{2v_o^2}{g}(\cos^2\theta - \sin^2\theta + 2\sin\theta\cos\theta\tan\alpha)$$

$$= \frac{2v_o^2}{g}(\cos 2\theta + \sin 2\theta \tan\alpha)$$

The derivative $\frac{dR_x}{d\theta} = 0$ when $\tan 2\theta = -\frac{1}{\tan\alpha} = -\cot\alpha$, which gives

$2\theta = 90^o + \alpha$, or $\underline{\theta_{max} = 45^o + \alpha/2}$.

(c) For $\alpha = 30^o$, $\theta_{max} = \underline{60^o}$.

(d) For $\theta = \theta_{max} = 60^o$ and $\alpha = 30^o$, we find the maximum range

$$R_{max} = \frac{2v_o^2}{g}\left(\frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{0.5774}{4}\right) = 0.5774\frac{v_o^2}{g}$$

Solving for v_o, we find the minimum speed required to attain a horizontal distance R_x

$$v_{min} = \sqrt{\frac{gR_x}{0.5774}}$$

For $R_x = 100$ m, we find that $v_{min} = \underline{41.2 \text{ m/s}}$.

<u>3-34</u>

The solution for Exercise 3-33 applies here if we substitute $\alpha = -\beta$. Then the horizontal distance R_x is given by

$$R_x = \frac{2v_o^2}{g}(\sin\theta\cos\theta + \tan\beta\cos^2\theta)$$

The distance is maximized for an elevation angle $\theta_{max} = 45^o - \beta/2$.

<u>3-35</u>

(a) Let $\vec{v}$ be the velocity of the point on the record rim with respect to the subway car. Using the notation provided in Fig. 3E-35, we have

$$\vec{v} = v_o[\hat{x}\cos\theta - \hat{y}\sin\theta]$$

Then $\vec{w} = u_o\hat{y} + \vec{v} = \hat{x}v_o\cos\theta + \hat{y}(u_o - v_o\sin\theta)$. In magnitude-and-direction form, we find

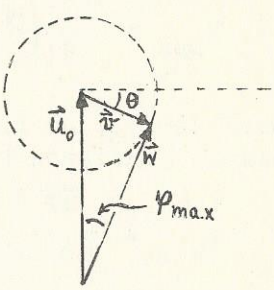
$$w = \sqrt{u_o^2 + v_o^2 - 2u_o v_o \sin\theta}$$

and w makes an angle of $\varphi = \sin^{-1}(v_o \cos\theta/w)$ with the y axis. (Here φ is measured clockwise from the y axis.)

(b) We determine v_o from $v_o = 2\pi r/T$, where T = (1/45) min = 1.33 s and r = 0.087 m. We find v_o = 0.41 m/s; we are also given u_o = 5.0 m/s. Therefore

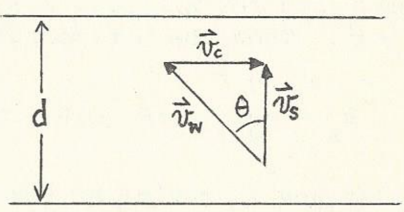
$$\vec{w} = [\hat{x}(0.41 \cos\theta) + \hat{y}(5.0 - 0.41 \sin\theta)] \text{ m/s}$$

Alternatively $\vec{w}$ has magnitude $\sqrt{25.2 - 4.1 \sin\theta}$ m/s and is directed at an angle of $\sin^{-1}(0.41 \cos\theta/w)$ with respect to the y axis. The speed w is largest when $\sin\theta = -1$, and is smallest when $\sin\theta = 1$. The extreme speeds are w_{max} = 5.4 m/s and w_{min} = 4.6 m/s. As shown in the figure at right, the absolute value of the angle φ is maximized when $\vec{v}$ is perpendicular to $\vec{w}$. Therefore $\varphi_{max} = \sin^{-1}(v_o/u_o)$ $= \sin^{-1}(0.41/5.0)$ = 4.69°. This occurs for $\theta = |\varphi|_{max}$ or $\theta = 180° - |\varphi|_{max}$, which gives θ = 4.69° or 175.3°. ($\varphi = |\varphi|_{max}$ when θ = 4.69°, and $\varphi = -|\varphi|_{max}$ when θ = 175.3°.



3-36

(a) Let $\vec{v}_c$ be the velocity of the current, $\vec{v}_w$ be the velocity of the swimmer with respect to the water and $\vec{v}_s$ be the velocity of the swimmer with respect to the shore. Then $\vec{v}_s = \vec{v}_w + \vec{v}_c$. To swim directly across, v_s must be perpendicular to the river bank as shown. Therefore $\sin\theta = v_c/v_w$, as shown in the figure. We are given the values v_c = 0.50 m/s, v_w = 0.70 m/s, and d = 50 m; we find θ = 45.6° upstream from the direction "straight across". The swimmer will increase her distance from the near shore at the rate $v_s = v_w \cos\theta$ = 0.49 m/s. She will cross the river in a time $t = d/v_s$ = 102 s.



(b) To maximize the component of her velocity perpendicular to the riverbank, the swimmer should head straight across the stream. She will cross in a time $t = d/v_w$ = 71.4 s, and she will land a distance $v_c t = (v_c/v_w)d$ = 35.7 m downstream from her starting point.