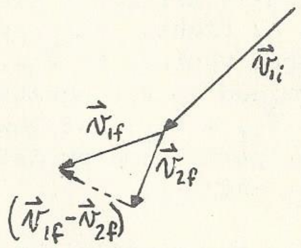
CHAPTER FOUR

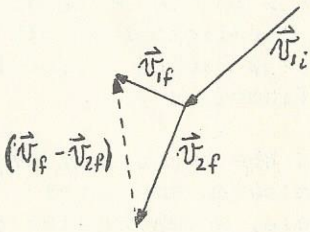
4-10

In the figure at right, the vectors $\vec{v}_{1i}$, $\vec{v}_{1f}$, and $\vec{v}_{2f}$ are shown, based on Figure 4-12. (The diagram has been doubled in size for clarity.) Also shown are the vectors $(\vec{v}_{1i} - \vec{v}_{2i}) = \vec{v}_{1i}$ and $(\vec{v}_{1f} - \vec{v}_{2f})$. Evidently $|\vec{v}_{1f} - \vec{v}_{2f}| < |\vec{v}_{1i} - \vec{v}_{2i}|$, showing that the collision is inelastic.



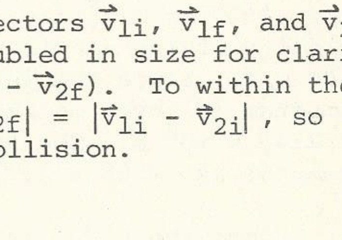
4-11

In the figure at right, the vectors $\vec{v}_{1i}$, $\vec{v}_{1f}$, and $\vec{v}_{2f}$ are shown, based on Figure 4-14. (The diagram has been doubled in size for clarity.) Also shown are the vectors $(\vec{v}_{1i} - \vec{v}_{2i}) = \vec{v}_{1i}$ and $(\vec{v}_{1f} - \vec{v}_{2f})$. To within the accuracy of the drawing, we have $|\vec{v}_{1f} - \vec{v}_{2f}| = |\vec{v}_{1i} - \vec{v}_{2i}|$, so the data are consistent with an elastic collision.



4-12

Let $\vec{v}_{1i}$ be the initial velocity of the incident particle, $\vec{v}_{2i} = \vec{0}$ be the initial velocity of the struck particle, and $\vec{v}_{1f}(\vec{v}_{2f})$ be the final velocity of the incident (struck) particle. If we use the strobe interval in Figure 4-20 as the unit of time, then we obtain the vectors $\vec{v}_{1i}$, $\vec{v}_{1f}$, and $\vec{v}_{2f}$ shown at right. (The diagram has been doubled in size for clarity.) Also shown are $(\vec{v}_{1i} - \vec{v}_{2i}) = \vec{v}_{1i}$ and $(\vec{v}_{1f} - \vec{v}_{2f})$. To within the accuracy of the drawing, we find $|\vec{v}_{1f} - \vec{v}_{2f}| = |\vec{v}_{1i} - \vec{v}_{2i}|$, so the data are consistent with an elastic collision.

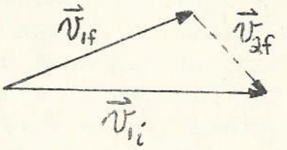


4-25

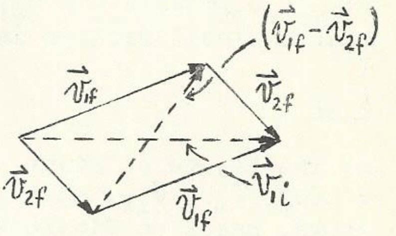
(a) Since the balls have equal mass m, the law of momentum conservation reads $m\vec{v}_{1i} + m\vec{v}_{2i} = m\vec{v}_{1f} + m\vec{v}_{2f}$. Because $\vec{v}_{2i} = \vec{0}$, so this equation reduces to $\underline{\vec{v}_{1i} = \vec{v}_{1f} + \vec{v}_{2f}}$.

(b) Interpreted graphically, the vector equation $\vec{v}_{1i} = \vec{v}_{1f} + \vec{v}_{2f}$ implies that the three vectors form the sides of a triangle. Therefore they must lie in a plane: the plane of the triangle.

(c) A copy of Figure 4E-25 (at a slightly different size) is given at right. Also shown (as a dashed vector) is $\vec{v}_{2f}$, as determined by the equation $\vec{v}_{1i} = \vec{v}_{1f} + \vec{v}_{2f}$. At lower right is the parallelogram determined by $\vec{v}_{1f}$ and $\vec{v}_{2f}$.



(d) In the parallelogram shown, $\vec{v}_{1i}$ is a diagonal and is shown as a dashed vector.



(e) The vector difference $\vec{v}_{1f} - \vec{v}_{2f}$ is the other diagonal in the parallelogram. It is also shown as a dashed vector in the lower figure.

(f) In the event that $|\vec{v}_{1f} - \vec{v}_{2f}| = |\vec{v}_{1i}|$, the diagonals of the parallelogram are equal. In this case the parallelogram is a rectangle, which implies that $\vec{v}_{1f}$ and $\vec{v}_{2f}$ are mutually perpendicular.

<u>4-26</u>

(a) Since the jet plane is traveling at constant velocity, a frame of reference fixed to the plane is an inertial frame. We find the rate of change of the momentum of the plane's surroundings; by Newton's second law of motion, the thrust on the plane will be equal in magnitude and opposite in direction. Taking the forward direction as positive, the initial velocity of the fuel (with respect to the plane) is 0, while the final velocity is -500 m/s. Since the burning rate is 3 kg/s, the plane expels momentum at the rate -1500 kg m/s^2 = -1500 N in the form of fuel. The initial velocity of the air is -200 m/s, while its final velocity is -500 m/s. Since the intake rate is 80 kg/s, the rate at which momentum is invested in the air is given by (80 kg/s) x (-300 m/s) = -2.40 x 10^4 N. The total rate at which the plane is expelling momentum is -2.55 x 10^4 N, so the thrust is <u>2.55 x 10^4 N</u>.

(b) Since the plane is traveling horizontally at constant speed, the drag force must balance the thrust. The drag force is <u>-2.55 x 10^4 N</u>, where the minus sign indicates the direction.

4-27

(a) The magnitude mv_o of the momentum change is (0.145 kg)(40 m/s) = 5.8 kg·m/s. The average accelerating force is the average rate of momentum change, which equals (5.8 kg·m/s) ÷ (0.25 s), or 23 N.

(b) The ball's initial momentum mv_o = 5.8 kg m/s, and the retarding force causes the momentum to decrease at the rate of 200 kg·m/s^2, so the stopping time t = (5.8 kg·m/s) ÷ (200 kg m/s^2) = 2.9×10^{-2} s. The stopping distance $d = \frac{1}{2}v_o t$ = (0.5)(40 m/s)(2.9×10^{-2} s) = 0.58 m. This stopping distance is similar to those observed in baseball games so the assumed value of 200 N for the retarding force must be reasonably close to the actual value.

4-28

(a) Since the physicist (once airborne) moves with constant downward acceleration g, the maximum height h is related to the launch speed v by $h = v^2/2g$, so that $v = \sqrt{2gh}$.

(b) The average net upward force F_{net} equals the average rate of change of momentum:

$$F_{net} = mv_o/\Delta t = m\sqrt{2gh}/\Delta t$$

Since gravity exerts a constant downward force of magnitude mg, the floor must exert an average upward force F_{floor} given by

$$F_{floor} = F_{net} + mg = m[g + (\sqrt{2gh}/\Delta t)]$$

When these forces are expressed as multiples of the physicist's weight W = mg, we have

$$(F_{net}/mg) = \sqrt{2h/g}/\Delta t \quad \text{and}$$

$$(F_{floor}/mg) = 1 + (\sqrt{2h/g}/\Delta t)$$

(c) When numerical values are inserted, we find F_{net} = 1960 N = 2.86W and F_{floor} = 2650 N = 3.86W.

4-29

(a) Let the initial and final velocities of the smaller sphere be $\vec{v}_{1i}$ and $\vec{v}_{1f}$; let the initial and final velocities of the larger sphere be $\vec{v}_{3i}$ and $\vec{v}_{3f}$. Let the masses of the spheres be m and 3m. The conservation of momentum during the collision implies that $m\vec{v}_{1i} + 3m\vec{v}_{3i} = m\vec{v}_{1f} + 3m\vec{v}_{3f}$. Since $\vec{v}_{1i} = v_o\hat{x}$, $\vec{v}_{2i} = \vec{0}$, and the line of centers during the collision is parallel to the x axis, we have $mv_o\hat{x} = mv_{1fx}\hat{x} + 3mv_{3fx}\hat{x}$, or

$$v_o = v_{1fx} + 3v_{3fx}$$

Because the collision is perfectly elastic and because the initial and final velocities all lie along a single line, we have $\vec{v}_{1i} - \vec{v}_{3i} = -(\vec{v}_{1f} - \vec{v}_{3f})$, or

$$v_o = -v_{1fx} + v_{3fx}$$

Solving the displayed equations, we find $v_{3fx} = v_o/2$ and $v_{1fx} = -v_o/2$. That is,

$$\vec{v}_{1f} = -\tfrac{1}{2}v_o\hat{x} \quad \text{and} \quad \vec{v}_{3f} = \tfrac{1}{2}v_o\hat{x}$$

(b) Using the same notation as in part (a), the conservation of momentum equation becomes $-3mv_o\hat{x} = mv_{1fx}\hat{x} + 3mv_{3fx}\hat{x}$, or

$$-3v_o = v_{1fx} + 3v_{3fx}$$

Again, because the collision is perfectly elastic and the initial and final velocities all lie along a single line, we have $\vec{v}_{1i} - \vec{v}_{3i} = -(\vec{v}_{1f} - \vec{v}_{3f})$, or

$$v_o = -v_{1fx} + v_{3fx}$$

Solving the displayed equation, we find $v_{3fx} = -v_o/2$ and $v_{1fx} = -3v_o/2$. That is,

$$\vec{v}_{1f} = -\tfrac{3}{2}v_o\hat{x} \quad \text{and} \quad \vec{v}_{3f} = -\tfrac{1}{2}v_o\hat{x}$$

<u>4-30</u>

(a) Because $\vec{v}_{2i} = \vec{0}$, the conservation of momentum implies that

$$m_1\vec{v}_{1i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

Since the collision is elastic and the initial and final velocities all lie along a single line, we have

$$\vec{v}_{1i} = -\vec{v}_{1f} + \vec{v}_{2f}$$

Solving the displayed equations, we find

$$\vec{v}_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{1i} \quad \text{and}$$

$$\vec{v}_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{1i}, \quad \text{as desired.}$$

(b) If $m_1 = m_2$, $\vec{v}_{1f} = \vec{0}$ and $\vec{v}_{2f} = \vec{v}_{1i}$.

(c) If $m_1 < m_2$, then $\vec{v}_{1f}$ and $\vec{v}_{1i}$ are antiparallel (or, oppositely directed).

(d) If $m_1 \ll m_2$ then $\vec{v}_{1f} \simeq -\vec{v}_{1i}$ and $\vec{v}_{2f} \simeq 2(m_1/m_2)\vec{v}_{1i}$, so that $v_{2f} \ll v_{1i}$.

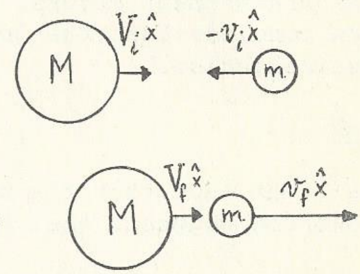
(e) If $m_1 > m_2$, then $\vec{v}_{1f}$ and $\vec{v}_{2f}$ are both parallel to (or, in the same direction as) $\vec{v}_{1i}$.

4-31

Denote the original direction of motion of the massive object by $\hat{x}$; let the final velocities of the massive object and the small object be $\vec{V}_f = V_f\hat{x}$ and $\vec{v}_f = v_f\hat{x}$, respectively. The conservation of momentum implies that $MV_i\hat{x} - mv_i\hat{x} = MV_f\hat{x} + mv_f\hat{x}$, or

BEFORE:

AFTER:



$$MV_i - mv_i = MV_f + mv_f$$

Because the collision is elastic and all velocities lie along a single line, we have $V_i\hat{x} - (-v_i\hat{x}) = -V_f\hat{x} + v_f\hat{x}$, or

$$V_i + v_i = -V_f + v_f$$

By multiplying the second displayed equation by M, and adding the result to the first displayed equation, we obtain $2MV_i + (M - m)v_i = (M + m)v_f$. Solving for v_f, we find

$$v_f = \left[\frac{(1 - m/M)}{(1 + m/M)}\right]v_i + \left[\frac{2}{(1 + m/M)}\right]V_i$$

Since $m \ll M$, $v_f \simeq v_i + 2V_i$, which was to be shown.

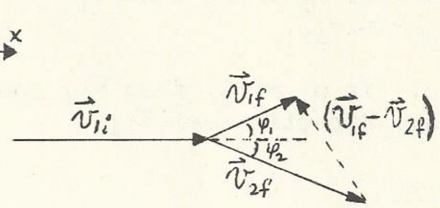
4-32

(a) The fact that the x-component of the velocity of puck 1 is not reversed, suggests that $m_1 > m_2$.

(b) We denote the initial velocities of pucks 1 and 2 by $\vec{v}_{1i} = v_{1i}\hat{x}$ and $\vec{v}_{2i} = \vec{0}$; the final velocities are $\vec{v}_{1f}$ and $\vec{v}_{2f}$. Letting the strobe interval be the time unit, and taking one centimeter as the distance unit, we obtain the following values from Figure 4E-32: $v_{1i} = 2.6$; $v_{1f} = 1.3$, and $v_{2f} = 2.3$. The velocity vectors are shown in the figure on the next page; the angles φ_1 and φ_2, as determined from Figure 4E-32, are 25.8^o and 21.5^o, respectively. The conservation of y-momentum implies that the mass ratio is given by

$$\frac{m_2}{m_1} = \frac{v_{1f}}{v_{2f}} \frac{\sin\varphi_1}{\sin\varphi_2} = \frac{1.3 \sin(25.8^\circ)}{2.3 \sin(21.5^\circ)} \simeq \underline{0.67}$$

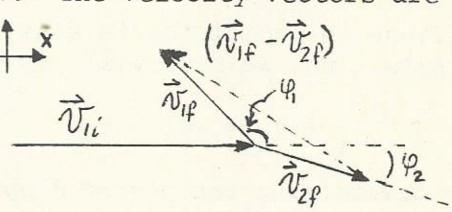
(c) From Figure 4E-32 (or from the figure at right), we obtain $|\vec{v}_{1f} - \vec{v}_{2f}| = 1.75$, while $|\vec{v}_{1i} - \vec{v}_{2i}| = v_{1i} = 2.6$. The difference between these measured values is definitely larger than the measurement errors, so we can conclude that the collision is inelastic.



4-33

(a) The fact that the x-component of the velocity of puck 1 is reversed suggests that $m_1 < m_2$.

(b) Using the notation and units described in the preceding solution, we obtain the following values from Figure 4E-33: $v_{1i} = 3.25$, $v_{1f} = 1.80$, and $v_{2f} = 1.57$. The velocity vectors are shown in the figure at the right; the angles φ_1 and φ_2, as determined from Figure 4E-33, are 134.7° and 16.5°, respectively. The conservation of y-momentum implies that the mass ratio is given by



$$\frac{m_2}{m_1} = \frac{v_{1f}}{v_{2f}} \frac{\sin\varphi_1}{\sin\varphi_2} = \frac{1.80 \sin(134.7^\circ)}{1.57 \sin(16.5^\circ)} \simeq \underline{2.9}$$

(c) From Figure 4E-33 (or from the figure at right), we obtain $|\vec{v}_{1f} - \vec{v}_{2f}| = |\vec{v}_{1i} - \vec{v}_{2i}|$ to within the accuracy of the drawing. That is, the data are consistent with an elastic collision.

4-34

Referring to Figure 4E-34, whether or not the second ball is placed at D, the ball C leaves the ramp traveling in a horizontal plane. So does the ball D when it is used. Therefore the flight times of balls C and D are the same. As a result the distances C'C" and D'D" have the same ratio as the absolute values of the "sideways" velocity components v'_y and v'_{Dy}. That is, C'C":D'D" $= |v'_{Cy}| : |v'_{Dy}|$. But the conservation of y-momentum requires that $m_C |v'_{Cy}| = m_D |v'_{Dy}|$. Therefore (m_C/m_D) = D'D":C'C" = $\underline{1/2}$.

4-35

(a) Let v_i be the speed with which the ball lands. The equations of motion for a freely falling object give $v_i = \sqrt{2gh_i}$ $= \sqrt{2(9.8)(1.0)} = \underline{4.43 \text{ m/s}}$.

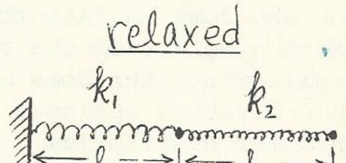
(b) Let v_f be the speed with which the ball rebounds. The free-fall equations give $v_f = \sqrt{2gh_f} = \sqrt{2(9.8)(0.5)} = \underline{3.13 \text{ m/s}}$.

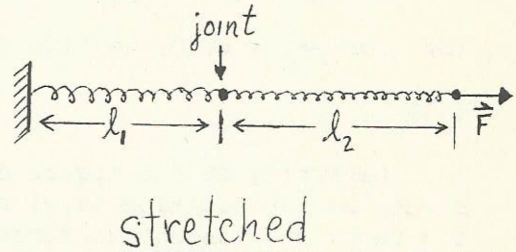
(c) The ball's velocity changed from 4.43 m/s downward to 3.13 m/s upward. Therefore its velocity changed by 7.56 m/s upward during 2.0×10^{-3} s. The average acceleration $\langle \vec{a} \rangle$ is given by $(\vec{v}_f - \vec{v}_i)/\Delta t$ $= \underline{3.78 \times 10^3 \text{ m/s}^2 \text{ upward}}$.

(d) Let $\vec{F}_D$ represent the force exerted by the disk. Since $\vec{F}_D = m\vec{a} - m\vec{g}$, we find the average force $\langle \vec{F}_D \rangle = m\langle \vec{a} \rangle - m\vec{g}$ $= (0.10)[3.78 \times 10^3 + 9.8] = \underline{3.79 \times 10^2 \text{ N upward}}$.

(e) The ratio of the magnitude of $\langle \vec{F}_D \rangle$ to the ball's weight equals $3.79 \times 10^2 \text{ N} \div [(0.10 \text{ kg})(9.8 \text{ m/s}^2] = \underline{3.87 \times 10^2}$.

4-36





The relaxed configuration of the springs is shown in the upper figure at right. To find the effective spring constant k', we suppose (for the sake of definiteness) that the springs are stretched and held motionless by an applied force $\vec{F}$, as shown in the lower figure. The rightward force exerted on the joint by spring #2 must equal $\vec{F}$, and must be balanced by the leftward force exerted on the joint by spring #1. Thus we have $F = k_1|l_1 - l_{1o}|$ $= k_2|l_2 - l_{2o}|$, where $l_1(l_{1o})$ and $l_2(l_{2o})$ are the actual (relaxed) lengths of the springs. Since both springs are stretched, the overall stretch $|l - l_o|$ is given by

$$|l - l_o| = |l_1 - l_{1o}| + |l_2 - l_{2o}|$$

where $l_o \equiv l_{1o} + l_{2o}$ is the relaxed length of the combination. The magnitude $S \equiv k'|l - l_o|$ of the restoring force must equal the magnitude F of the applied force. Therefore

$$k'|l - l_o| = k_1|l_1 - l_{1o}| = k_2|l_2 - l_{2o}|$$

Solving the displayed equations for k', we find $k' = k_1k_2/(k_1 + k_2)$.

(The same results obtain if we assume that the applied force $\vec{F}$ is compressive.)

4-37

Let ℓ_o be the equilibrium length of either spring. Because the springs are identical, each will stretch (or compress) by the same amount, provided that the externally applied force(s) acts (act) at the midpoint(s) of the bar(s), which we assume. (A symmetry argument can be used to establish this result: there is no need to invoke rotational mechanics.) If the instantaneous length of each spring is ℓ, then each spring exerts a restoring force of magnitude $k|\ell - \ell_o|$. Since these forces act in the same direction, the overall restoring force $S = 2k|\ell - \ell_o|$. The effective spring constant k" is defined by $S \equiv k''|\ell - \ell_o|$, so we have $k'' = 2k$.

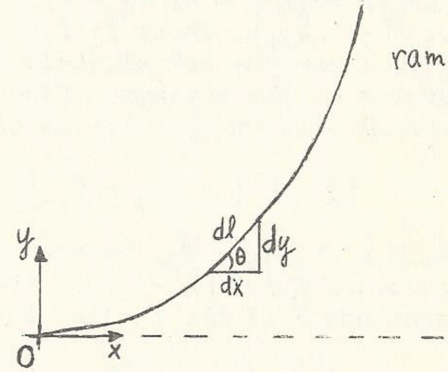
4-38

(a) If the astronomer is lying at angular distance α from the pole, then the dome surface must exert a normal force $N = W\cos\alpha$, where W is the weight of the astronomer plus blanket. To maintain static equilibrium, the dome must also exert a tangential force $T = W\sin\alpha$ directed upslope. The ratio of force magnitudes $T/N = \tan\alpha$ for equilibrium. But the ratio T/N cannot exceed the coefficient of static friction, μ_s. Therefore the maximum value of the tangent (and the corresponding maximum angle θ) is given by $\tan\theta = \mu_s$, or by $\theta = \tan^{-1}(\mu_s)$.

(b) For $\mu_s = 0.30$, we find $\theta = 16.7^o$.

4-39

(a) Referring to the figure at right, at any position (x,y) along the ramp, the tangential force T required to maintain equilibrium is given by $T = W\sin\theta = W(dy/d\ell)$ where W is the block's weight and $d\ell = \sqrt{(dx)^2 + (dy)^2}$. The required normal force $N = W\cos\theta = W(dx/d\ell)$. The condition that $T \leq \mu_s N$ therefore becomes $dy/d\ell \leq \mu_s(dx/d\ell)$, or $dy/dx \leq \mu_s$. Since $y = x^2/2L$, we find $dy/dx = x/L$. Thus $x_M/L = \mu_s$, or $x_M = \mu_s L$. The corresponding maximum height $y_M = \mu_s^2 L/2$.



(b) For L = 10 m and $\mu_s = 0.8$, we find x_M = <u>8 m</u> and y_s = <u>3.2 m</u>.

<u>4-40</u>

Using Eq. (4-29), the terminal speed of a falling body is determined by the equation $mg = \rho_f A \delta v_T^2/2$. Solving this equation for v_T, we find

$$v_T = \sqrt{\frac{2mg}{\rho_f A \delta}}$$

If this equation is applied to a sphere of radius r and density ρ then using $A = \pi r^2$ we find

$$v_T = \sqrt{\frac{8r\rho g}{3\rho_f \delta_s}}$$

where $\delta_s \simeq 0.4$ for a sphere. When this equation is compared with Eq. (4-28), we notice that the terminal velocity is much less sensitively dependent upon the size and density ($v_T \propto \sqrt{r\rho}$) when the flow is turbulent than it is when the resistive force is described by Stokes' law. In the latter case, v_T is proportional to the product $r^2\rho$. NOTE: Neither the above equation for terminal velocity nor Eq. (4-28) allows for buoyancy. If this is included, Eq. (4-28) becomes

$$v_T = \left(\frac{2r^2\rho g}{9}\right)(1 - \rho_f/\rho)$$

and the equation for turbulent flow is given by

$$v_T = \sqrt{\left(\frac{8r\rho g}{3\rho_f \delta_s}\right)\left(1 - \frac{\rho_f}{\rho}\right)}$$

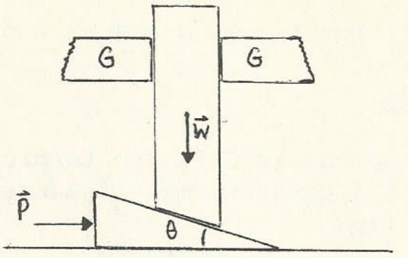
<u>4-41</u>

We apply $v_T = \sqrt{2mg/\rho_f A\delta}$, with $A = \pi r^2$, where r = 5 m. With the other given numerical values, we obtain $v_T \simeq$ <u>4.2 m/s</u>. Since the average density of the parachutist plus parachute is approximately one thousand times the density of the air, buoyancy can be neglected.

<u>4-42</u>

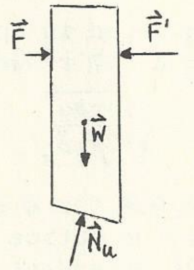
(a) Figure 4E-42 is redrawn at the top of the next page. Since all surfaces are highly lubricated, all contact forces are normal to the surface of contact. A free body diagram for the vertical block is also shown on the next page. Since the force(s) $\vec{F}$ and $\vec{F}'$ exerted by blocks G and G' are strictly horizontal, the vertical component of the normal force $\vec{N}_u$ exerted on the vertical block by the wedge must balance the weight, $\vec{W}$. This gives the equation

$$N_u \cos\theta = W$$



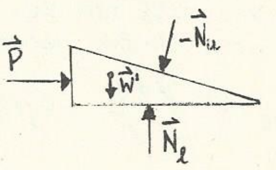
A free body diagram for the wedge is shown at bottom right. Since the weight $\vec{W}'$ and the force $\vec{N}_\ell$ exerted on the wedge by the floor are strictly vertical, the horizontal component of the force $-\vec{N}_u$ must balance the force $\vec{P}$. This gives the equation

$$N_u \sin\theta = P.$$



From the two displayed equations, we find that the rightward force $\vec{P}$ has magnitude <u>$P = W \tan\theta$.</u>

(b) As described in part (a), the horizontal component of the normal force $-\vec{N}_u$ (exerted on the wedge by the vertical block) balances the push $\vec{P}$.



(c) The sum $\vec{F} + \vec{F}'$ of the forces exerted on the vertical block by the guides G and G' is a leftward horizontal force which balances the horizontal component of $\vec{N}_u$.

<u>4-43</u>

(a) Let v_g and v_c be the velocity of the girl with respect to the ground and the velocity of the cart with respect to the ground, respectively. Take the rightward direction to be positive. The velocity of the girl with respect to the cart is given by $v_g - v_c$. Since momentum is conserved, we have $m_g v_g + m_c v_c = \text{constant} = 0$. With numerical values $v_g - v_c = 2$ m/s, $m_g = 50$ kg, and $m_c = 100$ kg, we find $v_c =$ <u>$-2/3$ m/s</u>.

(b) Since $v_g = v_c + 2$ m/s, we have $v_g =$ <u>$4/3$ m/s</u>.

(c) The time required is the distance divided by the speed: 4 m ÷ 2 m/s = <u>2 s</u>.

(d) During 2 s, the girl travels (2 s)(4/3 m/s) = <u>8/3 m</u>.

(e) The cart travels (2 s)(-2/3 m/s) = <u>-4/3 m</u>, where the minus sign indicates leftward motion.

(f) Since momentum conservation implies that $m_g v_g + m_c v_c = 0$, the cart's speed (and the girl's) must vanish if $v_g = v_c$.

<u>4-44</u>

(a) Since the girl comes to a halt with respect to the cart just after stepping up, we can readily apply the conservation of momentum to write

$$m_g v_{go} = (m_g + m_c) v_f$$

where $v_{go} = 2$ m/s and v_b is the common velocity of girl and cart with respect to the ground. Using the given mass values, we find $v_b = \underline{2/3 \text{ m/s}}$.

(b) Until the girl steps off again, the momentum of the system "girl plus cart" is conserved, so we have $m_g v_g' + m_c v_c' = (m_g + m_c) v_f$. We are given that $v_g' - v_c' = 2$ m/s. Using this equation and the previous one, we find that $v_c' = 0$. The cart does not move while the girl walks across it.

(c) The girl's velocity $v_g' = \underline{2 \text{ m/s}}$ with respect to the ground.

(d) The time T required is 4 m $\div$ 2 m/s = <u>2 s</u>.

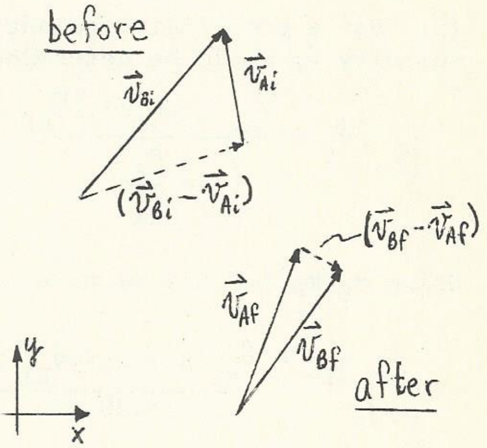
(e) The girl moves a distance $v_g' T = (2 \text{ m/s})(2 \text{ s}) = \underline{4 \text{ m}}$.

(f) The cart moves a negligible distance during the entire process. (It moves slightly to the right during the brief interval while the girl is standing at the left end of the cart.)

(g) The velocity of the cart is zero when the girl steps down, and remains <u>zero</u> thereafter.

<u>4-45</u>

(a) The figures at right show the initial and final velocities of bodies A and B. The initial and final relative speeds $|\vec{v}_{Bi} - \vec{v}_{Ai}|$ and $|\vec{v}_{Bf} - \vec{v}_{Af}|$ could be evaluated from these figures, but we give an analytical evaluation. Applying the law of cosines to the triangle in the lower figure,



we have

$$|\vec{v}_{Bf} - \vec{v}_{Af}| = \sqrt{v_{Bf}^2 + v_{Af}^2 - 2v_{Bf}v_{Af}\cos(\varphi_{Bf} - \varphi_{Af})}$$

$$= \sqrt{(16)^2 + (16)^2 - (2)(16)(16)\cos(-15.4^\circ)}$$

$$= \underline{4.29 \text{ m/s}}$$

An analogous expression applies to the upper figure, giving

$$|\vec{v}_{Bi} - \vec{v}_{Ai}| = \sqrt{(20)^2 + (10)^2 - (2)(20)(10)\cos(-60^\circ)}$$

$$= \sqrt{300} = \underline{17.3 \text{ m/s}}$$

Therefore $|\vec{v}_{Bf} - \vec{v}_{Af}| / |\vec{v}_{Bi} - \vec{v}_{Ai}| = 0.248 < 1$; the collision was <u>not</u> elastic.

(b) The law of conservation of momentum implies that $m_A(\vec{v}_{Af} - \vec{v}_{Ai}) = -m_B(\vec{v}_{Bf} - \vec{v}_{Bi})$. Therefore

$$\frac{m_B}{m_A} = -\frac{(\vec{v}_{Af} - \vec{v}_{Ai})\cdot\hat{x}}{(\vec{v}_{Bf} - \vec{v}_{Bi})\cdot\hat{x}} = -\frac{(\vec{v}_{Af} - \vec{v}_{Ai})\cdot\hat{y}}{(\vec{v}_{Bf} - \vec{v}_{Bi})\cdot\hat{y}}$$

The velocity change components along $\hat{x}$ are $-(\vec{v}_{Af} - \vec{v}_{Ai})\cdot\hat{x} = -[16\cos(70^\circ) - 10\cos(110^\circ)] = -8.893$ and $(\vec{v}_{Bf} - \vec{v}_{Bi})\cdot\hat{x} = 16\cos(54.6^\circ) - 20\cos(50^\circ) = -3.587$. Therefore $m_B/m_A = (-8.893)/(-3.587) = 2.479$. The velocity change components along $\hat{y}$ are $-(\vec{v}_{Af} - \vec{v}_{Ai})\cdot\hat{y} = -[16\sin(70^\circ) - 10\sin(110^\circ)] = -5.638$ and $(\vec{v}_{Bf} - \vec{v}_{Bi})\cdot\hat{y} = 16\sin(54.6^\circ) - 20\sin(50^\circ) = -2.279$. Therefore $m_B/m_A = (-5.638)/(-2.279) = 2.474$. The two results for the mass ratio are consistent to within the precision of the given numerical values; averaging them, we obtain $m_B/m_A = \underline{2.48}$.

(c) For a completely inelastic collision, the common final velocity $\vec{v}_f$ would be determined by $m_A\vec{v}_{Ai} + m_B\vec{v}_{Bi} = (m_A + m_B)\vec{v}_f$, or

$$\vec{v}_f = \frac{\vec{v}_{Ai} + (m_B/m_A)\vec{v}_{Bi}}{\left(1 + \frac{m_B}{m_A}\right)}$$

Using $m_B/m_A = 2.48$, we have

$$v_{fx} = \frac{\vec{v}_{Ai}\cdot\hat{x} + 2.48\vec{v}_{Bi}\cdot\hat{x}}{3.48} \qquad \text{or}$$

$$v_{fx} = \frac{10\cos(110^\circ) + (2.48)(20)\cos(50^\circ)}{3.48}$$

$$= 8.179 \text{ m/s} \simeq 8.18 \text{ m/s}$$

The component along $\hat{y}$ is similarly given by

$$v_{fy} = \frac{10\sin(110^\circ) + (2.48)(20)\sin(50^\circ)}{3.48}$$

$$= 13.62 \simeq 13.6 \text{ m/s}$$

Thus we have $\vec{v}_f = \underline{(8.18 \text{ m/s})\hat{x} + (13.6 \text{ m/s})\hat{y}}$. If we express this result as a magnitude and direction, we find $v_f = \underline{15.9 \text{ m/s}}$ and $\varphi_f = \tan^{-1}(13.62/8.179) = \underline{59.0^\circ}$.

4-46

Referring to Figure 4E-46 and letting $\hat{x}$ be rightward and $\hat{y}$ upward, the conservation of momentum along $\hat{y}$ gives

$$0 = m_B v_{Bf} \sin(68^\circ) - m_A v_{Af} \sin(8.5^\circ), \quad \text{or}$$

$$m_B = \frac{m_A v_{Af} \sin(8.5^\circ)}{v_{Bf} \sin(68^\circ)}$$

Since we are given m_A = 4 unified mass units = 4 u, and $v_{Bf}/v_{Af} \simeq 0.63$, we obtain

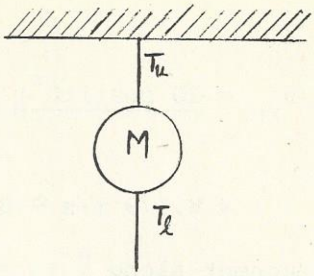
$$m_B \simeq \frac{4\sin(8.5^\circ)}{(0.63)\sin(68^\circ)} \simeq \underline{1.01 \text{ u}}.$$

This mass value is consistent with either a proton or a neutron. Since the particle left a track in the cloud chamber, it cannot be a neutron. Thus we infer that it is a proton. (NOTE: In the first printing of the text, the numerical value of the speed ratio was incorrectly given as $v_{Bf}/v_{Af} \simeq 0.68$. This leads to a mass $m_B \simeq 0.94$ u, which is not consistent with the mass of any long-lived nuclear particle.)

4-47

(a) Using the figure at the top of the next page, if downward is taken as the positive direction, then Newton's second law reads $Mg + T_\ell - T_u = Ma$, or $T_u = T_\ell + M(g - a)$.

(b) First experiment: When the lower thread is pulled with a



very gradually increasing force (T_ℓ increases slowly), so that $a \ll g$, then $T_u \simeq T_\ell + Mg$. That is, when the pull is very gradually increased, T_u exceeds T_ℓ at each instant. If the threads are identical, the upper thread reaches its breaking tension first.
Second experiment: When the lower thread is jerked downward, so that (for a brief instant) $a \gg g$, we find $T_u = T_\ell + M(g - a) < T_\ell$. That is, the tension in the lower thread exceeds that in the upper one. If one of the threads breaks, it must be the lower one. (NOTE: In thinking about this problem, it is helpful to regard the threads as massless but slightly extensible, with the instantaneous tension a strictly increasing function of the instantaneous thread length.)

4-48

(a) The free-fall time Δt is determined by $h = \frac{1}{2}g(\Delta t)^2$, which gives $\Delta t = \sqrt{2h/g}$.

(b) The mass Δm of the falling stream is given by $\Delta m = c\Delta t = c\sqrt{2h/g}$.

(c) The weight ΔW of the stream is given by $\Delta W = (\Delta m)g = c\sqrt{2gh}$.

(d) The speed v at impact is given by $v = g\Delta t = \sqrt{2gh}$; the direction is downward.

(e) Assuming that the sand comes to rest without rebounding at impact, the momentum per unit time given to the bottom of the vessel equals $cv = c\sqrt{2gh}$ downward.

(f) The downward force F is identically equal to the rate at which momentum is transferred: $F = c\sqrt{2gh}$.

(g) Once the sand is flowing at a steady rate, the total mass M of sand consists of a mass $\Delta m = c\sqrt{2h/g}$ in the falling stream and a mass $M - \Delta m$ which is motionless (having either not fallen yet or having already landed). The total downward force exerted by the sand is the weight $(M - \Delta m)g$ of supported sand, plus the force $F = cv = c\sqrt{2gh}$ being exerted by the braking sand stream. Because $c\sqrt{2gh} = c\sqrt{2h/g}\cdot g = (\Delta m)g$, the total force is $(M - \Delta m)g + (\Delta m)g = Mg$. But Mg is the initial weight force against which the system was initially balanced (before the stopcock was opened).

(h) If the system is left unclamped, then as the supply of sand is exhausted, the arm supporting the vessel dips down. This happens because more sand is landing at B than is being released at A.

(The sum of the supported weight and the momentum-transfer force F temporarily exceeds Mg.)

4-49

(a) Let F_A be the magnitude of the frictional force along interface A; it is a rightward force on block 1. Let F_B be the frictional force magnitude along interface B, it is a rightward force on block 2. Let N_A and N_B be the normal forces along interfaces A and B, with positive values representing repulsive forces ("pushes"). Applying Newton's second law to block 1, we obtain

$$F_A = m_1 a \qquad \text{and}$$

$$N_A = m_1 g$$

Since the pairs exerted between blocks 1 and 2 cancel (by Newton's third law) and since there is no slippage, we can apply Newton's second law to the composite object "blocks 1 and 2". The horizontal and vertical components of Newton's second law are

$$F_B \cos\theta_B - N_B \sin\theta_B = (m_1 + m_2)a \qquad \text{and}$$

$$F_B \sin\theta_B + N_B \cos\theta_B = (m_1 + m_2)g$$

Solving these for F_B and N_B, we find

$$F_B = (m_1 + m_2)(a \cos\theta_B + g \sin\theta_B) \qquad \text{and}$$

$$N_B = (m_1 + m_2)(g \cos\theta_B - a \sin\theta_B)$$

For surface A, the ratio of the static frictional force to its maximum value $F_{A,max} = \mu_A N_A$ is given by

$$\frac{F_A}{F_{A,max}} = \frac{a}{\mu_A g}$$

For surface B, the ratio is

$$\frac{F_B}{F_{B,max}} = \frac{F_B}{\mu_B N_B} = \frac{a \cos\theta_B + g \sin\theta_B}{\mu_B (g \cos\theta_B - a \sin\theta_B)}$$

$$= \frac{1}{\mu_B}\left[\frac{(a/g) + \tan\theta_B}{1 - (a/g)\tan\theta_B}\right]$$

(b) As the acceleration a is increased, the ratios found in part (a) increase. When one of the ratios reaches unity, slippage will occur along the corresponding interface. Slippage will occur along surface A when $a/g > \mu_A$. Slippage will occur along surface B when

$$\mu_B\left[1 - (a/g)\tan\theta_B\right] > (a/g) + \tan\theta_B \quad \text{or}$$

$$\frac{a}{g} > \frac{\mu_B - \tan\theta_B}{1 + {}_B\tan\theta_B}$$

Therefore interface A slips first if

$$\mu_A < \frac{\mu_B - \tan\theta_B}{1 + \mu_B\tan\theta_B}$$

If the inequality is reversed, surface B slips first.

(c) The equations of part (b) apply with the substitution $\theta_B \rightarrow -\theta_B$ (and with the understanding that the directions of the frictional forces are reversed). Therefore interface A slips first if

$$\mu_A < \frac{\mu_B + \tan\theta_B}{1 - \mu_B\tan\theta_B}$$

If the inequality is reversed, surface B slips first.

(d) In order that the slippage be along B for rightward acceleration and along A for leftward acceleration, we need

$$\frac{\mu_B - \tan\theta_B}{1 + \mu_B\tan\theta_B} < \mu_A < \frac{\mu_B + \tan\theta_B}{1 - \mu_B\tan\theta_B}$$

For given values of μ_A and μ_B, this requires

$$\tan\theta_B > \max\left(\frac{\mu_B - \mu_A}{1 + \mu_A\mu_B}, \frac{\mu_A - \mu_B}{1 + \mu_A\mu_B}\right) = \left|\frac{\mu_A - \mu_B}{1 + \mu_A\mu_B}\right|$$

For $\mu_A = 0.50$ and $\mu_B = 0.80$, we need $\theta_B > \tan^{-1}\left(\frac{0.3}{1.4}\right) \simeq 12.1^\circ$

However, we also need $\theta_B \leq \tan^{-1}(\mu_B) \simeq 38.7^\circ$, in order that the blocks can remain stacked at rest with slipping. Thus we have $\underline{12.1^\circ < \theta_B \leq 38.7^\circ}$.

4-50

(a) The normal force exerted on the block by the incline is $N = mg\cos\alpha$. Once in motion, the net force downward along the incline is mg $\sin\alpha - \mu_k mg\cos\alpha$, so the acceleration $a = g(\sin\alpha - \mu_k \cos\alpha)$. It will strike the second block after a time t_1 determined by $\ell = \frac{1}{2}at_1^2$, so $t_1 = \sqrt{2\ell/a}$. The impact speed $v_1 = at_1 = \sqrt{2a\ell}$.

(b) Let v_{ji} and v_{jf} denote the initial and final speeds of the jth block. Because the collisions are elastic and the blocks have equal masses, $v_{ji} = v_{j-1,f}$. Each block slides a distance ℓ with acceleration a, so that $v_{jf}^2 = v_{ji}^2 + 2a\ell$. Using these results and recognizing that $v_{1i} = 0$, we find $v_{ji} = \sqrt{2(j-1)a\ell}$ and $v_{jf} = \sqrt{2ja\ell}$. The total elapsed time T is the sum of the elapsed times between collisions. (The blocks are assumed to be perfectly rigid, so the collisions themselves take no time.) Letting t_j be the sliding time of block j, we have $t_j = \ell/\langle v_j\rangle = 2\ell/(v_{jf} + v_{ji})$. Then we have

$$T = \sum_{j=1}^{n} t_j = 2\ell \sum_{j=1}^{n}\left(\frac{1}{\sqrt{2ja\ell} + \sqrt{2(j-1)a\ell}}\right)$$

$$= \sqrt{\frac{2\ell}{a}} \sum_{j=1}^{n} \frac{1}{(\sqrt{j} + \sqrt{j-1})} = \sqrt{\frac{2\ell}{a}} \sum_{j=1}^{n} (\sqrt{j} - \sqrt{j-1})$$

$$= \sqrt{\frac{2n\ell}{a}}$$

The complicated solution just given can be avoided by recognizing that the problem is equivalent to the problem of a single block traveling a distance $n\ell$ at acceleration a: the elapsed time T is just the time required for a block to traverse the open distance $n\ell$ when accelerated from rest with acceleration a.

(c) A solitary block will traverse a distance s in a time $T' = \sqrt{2\cdot s/a}$. In the present case $s = n\ell + (n-1)d$. Therefore

$$T' = \sqrt{\frac{2n\ell}{a}}\sqrt{1 + \frac{(n-1)d}{n\ell}} > T$$

The single block requires a longer time to reach the bottom end of the ramp. The final speed v' is given by

$$v' = \sqrt{2na\ell}\sqrt{1 + \frac{(n-1)d}{n\ell}} > v_{nf}$$

The final speed v' of the solitary block is greater than the final speed of the nth.

(d) Applying the conservation of momentum in the case of completely inelastic collisions, we find $v''_{ji} = (j-1)v''_{j-1,f}/j$. As before, $(v''_{jf})^2 = (v''_{ji})^2 + 2a\ell$.

From these we find the recurrence relation

$$\left(v''_{ji}\right)^2 = \frac{(j-1)^2}{j^2}\left[\left(v''_{j-1,i}\right)^2 + 2a\ell\right]$$

When the condition $v''_{1i} = 0$ is applied, we find

$$\left(v''_{ji}\right)^2 = \frac{2a\ell}{j^2}\sum_{k=1}^{j} k^2$$

The sum in the above equation is listed in most mathematical handbooks; it has the value $[(j-1)j(2j-1)/6]$. Therefore

$$\left(v''_{ji}\right)^2 = 2a\ell\left[\frac{(j-1)(2j-1)}{6j}\right] \qquad \text{or}$$

$$v''_{ji} = \sqrt{2a\ell}\sqrt{\frac{(j-1)(2j-1)}{6j}}$$

From this we proceed to obtain

$$v''_{jf} = \sqrt{(v''_{ji})^2 + 2a\ell} = \sqrt{2a\ell}\sqrt{\frac{(j+1)(2j+1)}{6j}}$$

Using $t''_j = 2\ell/(v''_{jf} + v''_{ji})$, we find

$$\begin{aligned} T'' &= \sum_{j=1}^{n} t''_j \\ &= \sqrt{\frac{2\ell}{a}} \cdot \sum_{j=1}^{n}\left[\sqrt{\frac{(j+1)(2j+1)}{6j}} + \sqrt{\frac{(j-1)(2j-1)}{6j}}\right]^{-1} \\ &= \sqrt{\frac{2\ell}{a}} \cdot \sum_{j=1}^{n}\left[\sqrt{\frac{(j+1)(2j+1)}{6j}} - \sqrt{\frac{(j-1)(2j-1)}{6j}}\right] \end{aligned}$$

(e) By making a term-by-term comparison, it can be shown that $T'' > T$. However, T'' may be greater than or less than T' of part (c), depending on the numerical value of d/ℓ. The final speed

$$v''_{nf} = \sqrt{2a\ell}\sqrt{\frac{(n+1)(2n+1)}{6n}} < \sqrt{2na\ell},$$

so $v''_{nf} < v_{nf} < v'$. To summarize, the case of completely inelastic collisions leads to the lowest final speed, and requires a time longer than that for elastic collisions. However, the time required in the completely inelastic case may be greater than or less than the overall sliding time for a solitary block, depending on the value of the ratio d/ℓ.

4-51

(a) The maximum deceleration that static friction can provide is $\mu_s g$. Therefore the minimum stopping distance that avoids slippage is $v_o^2/2\mu_s g$. The trucker can stop only if $S_o \geq v_o^2/2\mu_s g$.

(b) For the given values, $S_o = 100$ m $\geq v_o^2/2\mu_s g = 91.8$ m, so the trucker can stop without causing slippage.

(c) During the trucker's reaction time Δt, the truck travels a distance $v_o\Delta t$. Therefore the condition for stopping without slippage becomes $S_o \geq v_o\Delta t + (v_o^2/2\mu_s g)$.

(d) For $v_o = 30$ m/s and $\Delta t = 0.5$ s, $v_o\Delta t + (v_o^2/2\mu_s g)$ $= 15$ m $+ 91.8$ m > 100 m, so the trucker cannot stop without slippage.

CHAPTER FIVE

5-9

(a) Before release, the ball moves in a circle relative to the ground. When the centripetal force constraining it to that circle is removed, the ball moves in a straight line tangent to the circle at the point of release.

(b) The child sees the ball moving outward due to the fictitious centrifugal force and curving to the right (that is, backwards). The latter effect is due to the fictitious Coriolis force.

5-12

(a) No

(b) No

(c) The effective gravitational field $\vec{g}'$ inside the train is given by $\vec{g}' = \vec{g} - \vec{a}$, where $\vec{a}$ is the train's acceleration. Since the train is windowless, the direction of the earth's gravitational field $\vec{g}$ with respect to the train is completely unknown. Therefore, although it is possible to find the magnitude and direction of $\vec{g}'$ with reference to the train, it is not possible to solve for the train's acceleration $\vec{a}$. (If $|\vec{g}'| \neq |\vec{g}|$, it is possible to conclude that $\vec{a} \neq \vec{0}$.)

5-13

A reasonable estimate for the radius R of the swing's path is 3 m;