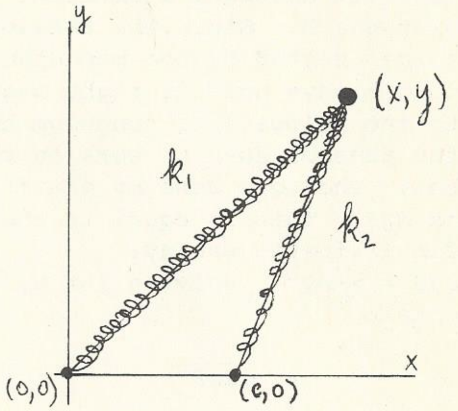
7-17



The net force $\vec{F}$ exerted by the two springs is given by $\vec{F} = \vec{F}_1 + \vec{F}_2$, where $\vec{F}_1$ and $\vec{F}_2$ are the forces exerted by spring #1 and spring #2, respectively. Since the springs are assumed to have negligible relaxed lengths, we have, referring to the figure, $\vec{F}_1 = -k_1[\hat{x}x + \hat{y}y]$ and $\vec{F}_2 = -k_2[\hat{x}(x - c) + \hat{y}y]$. Therefore the components of $\vec{F}$ are given by

$$F_x = F_{1x} + F_{2x} = -k_1x - k_2(x - c)$$
$$= -(k_1 + k_2)x + k_2c, \quad \text{and}$$
$$F_y = F_{1y} + F_{2y} = -(k_1 + k_2)y$$

These agree with the expressions obtained in Example 7-11.

7-18

We denote the mass of the puck by m and its initial velocity by $v\hat{x}$. If a time Δt is required for it to come to rest under constant acceleration, a force $\vec{F}_p = \hat{x}(m\Delta v_x/\Delta t) = \hat{x}(-mv/\Delta t)$ must be exerted on the puck. Newton's third law implies that the puck exerts a force $\vec{F} = -\vec{F}_p = \hat{x}(mv/\Delta t)$ on the nail. The work W done by this force is given by $W = F_x\Delta x$, where $\Delta x = \overline{v_x}\Delta t = \frac{1}{2}v\Delta t$. Therefore the work done is given by

$$W = \left(\frac{mv}{\Delta t}\right)\left(\frac{1}{2}v\Delta t\right) = \frac{1}{2}mv^2$$

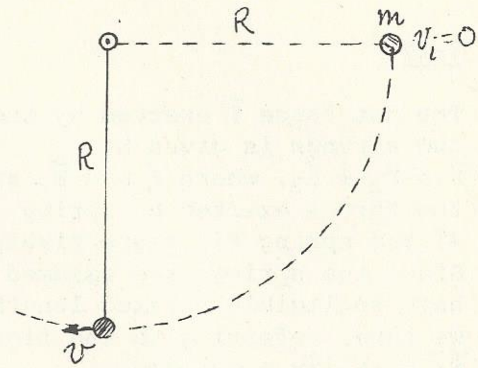
This agrees with Eq. (7-4), which gives the work done on the puck when it is accelerated from rest to speed v.

7-19

The work done in raising the puck from x = 0 to x = x' is $W_1 = mgx'$. The work done in lowering the puck from x = x' to $x < x'$ is $W_2 = -mg|x - x'| = mg(x - x')$. Therefore the total work done in the process (which raises the puck from 0 to x) is given by $W = W_1 + W_2 = mgx' + mg(x - x') = \underline{mgx}$.

7-20

As shown in the figure, the pendulum descends a vertical distance R. Since the tension force exerted by the pendulum cord always acts at right angles to the velocity of pendulum bob, the tension does no work on the bob. The work done by gravity is mgR. This is equal to the final kinetic energy: $mgR = \frac{1}{2}mv^2$. Solving for v, we obtain



$$v = \sqrt{2gR}$$

With R = 0.5 m, we find $v = \sqrt{2(9.8)(0.5)} = \underline{3.13\ m/s}$.

7-21

(a) The loss of potential energy as body B descends a distance D is given by m_BgD. This must equal the gain in kinetic energy of the system. Denoting the (common) speed by v, we find $\frac{1}{2}(m_A + m_B)v^2 = m_BgD$, so that

$$v = \sqrt{\frac{2m_BgD}{m_A + m_B}}$$

(b) Using the values m_A = 20 kg, m_B = 30 kg, and D = 5.0 m, we find v = $\underline{7.67\ m/s}$.

7-22

We begin by remarking that the midpoint of the stick is presumed to be fixed to the pivot point, so that both of the bodies must travel along a circle of radius $\ell/2$, centered at the pivot. (Without this constraint, the center of the rod would begin to move toward the original position of the heavier object as the rod starts to turn.) Under this assumption, as the heavier body passes under the lighter one, the system's potential energy has been changed by an amount $U_f - U_i = -2mg(\ell/2) + mg(\ell/2) = -mg\ell/2$. The corresponding change in kinetic energy is given by

$$K_f - K_i = -(U_f - U_i) = mg\ell/2$$

Because the system started from rest ($v_i = 0$) and since both objects (total mass 3m) move with the same speed, we find that

$$\frac{1}{2}(3m)\cdot v_f^2 = mg\ell/2$$

Solving for v_f, we obtain $\underline{v_f = \sqrt{g\ell/3}}$.

7-23

We recall that the maximum range for a projectile launched with speed v_o from ground level is given by $R_{max} = v_o^2/g$. We use this expression since no information is given on the launch elevations. The initial kinetic energy $K_o \equiv \frac{1}{2}mv_o^2 = \frac{1}{2}mgR_{max}$. We apply this expression, with $g = 9.80\ m/s^2$, to the various cases given.

(a) Shot put: $m = 7.26$ kg and $R_{max} = 22.0\ m \Rightarrow K_o = 783$ J

(b) Discus throw: $m = 2.00$ kg and $R_{max} = 70.9\ m \Rightarrow K_o = 695$ J

(c) Hammer throw: $m = 7.26$ kg and $R_{max} = 79.3\ m \Rightarrow K_o = 2.82 \times 10^3$ J

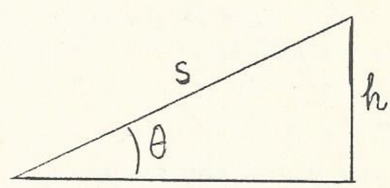
(d) Javelin throw: $m = 0.800$ kg and $R_{max} = 94.6\ m \Rightarrow K_o = 371$ J

(e) Long jump: $m = 60.0$ kg and $R_{max} = 8.90\ m \Rightarrow K_o = 2.62 \times 10^3$ J

(f) Baseball throw: $m = 0.145$ kg and $R_{max} = 130\ m \Rightarrow K_o = 92.4$ J

7-24

The incline is shown at right. If the block travels the entire length s of the incline, the frictional force will perform work $-\mu_k Ns$ on the block. Furthermore, the gravitational potential energy of the block will increase by mgh. To reach the top, the block must have initial kinetic energy $\frac{1}{2}mv_o^2 \geq mgh + \mu_k Ns$. Since $N = mg\cos\theta$ and $h = s\sin\theta$, we have



$$\frac{1}{2}mv_o^2 \geq mgs[\sin\theta + \mu_k\cdot\cos\theta].$$

With $\theta = 20^o$, $m = 2.0$ kg, $s = 3.0$ m, and $\mu_k = 0.40$, we find $(\frac{1}{2}mv_o^2)_{min} = (2.0)(9.8)(3.0)[(0.342) + (0.40)(0.940)] = \underline{42.2\ J}$.

7-25

(a) We denote the initial and final masses by m_i and m_f, and the corresponding rocket velocities by V_i and V_f. We integrate the given equation:

$$\int_{V_i}^{V_f} dV = \int_{m_i}^{m_f} v'_g \frac{dm}{m}$$

This yields

$$V_f - V_i = v'_g[\ln(m_f) - \ln(m_i)] = v'_g \ln\left(\frac{m_f}{m_i}\right)$$

which is the desired equation.

(b) Since the rocket starts from rest, we have $V_i = 0$. To attain a final speed $V_f = 1.5|v'_g|$, we must have

$$1.5|v'_g| = v'_g \ln\left(\frac{m_f}{m_i}\right)$$

Since $v'_g < 0$, we obtain $\ln(m_f/m_i) = -1.5$, or $m_f = m_i e^{-1.5} = \underline{0.223 m_i}$.

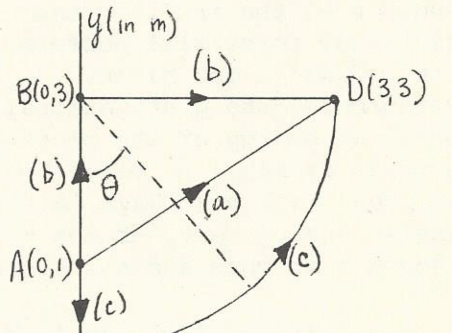
7-26

(a) In order to bring the automobile to rest, the frictional force must perform work $-K \equiv -\frac{1}{2}mv^2$. Assuming that the automobile just barely avoids skidding, the work done by the frictional force while the braking automobile travels a distance s is given by $-\mu_s mgs$. Letting s_{min} denote the minimum stopping distance, we have $-\frac{1}{2}mv^2 = -\mu_s mgs_{min}$ or $\underline{s_{min} = v^2/2\mu_s g}$.

(b) With $v_o = 30$ m/s, $\mu_s = 0.5$, and $g = 9.8$ m/s^2, we find $s_{min} = \underline{91.8 \text{ m}}$.

7-27

We are given the force $\vec{F}_1 = 2x^2\hat{x} + 3y^2\hat{y}$, and the endpoints A = (0,1) and D = (3,3), as shown in the figure at right. The force is given in newtons and the coordinates are given in meters. The work W (in joules) done by $\vec{F}_1$ as a particle moves along any path from A to D is given by



$$W = \int_A^D \vec{F}_1 \cdot d\vec{s} = \int_{(0,1)}^{(3,3)} (2x^2\hat{x} + 3y^2\hat{y}) \cdot (\hat{x}dx + \hat{y}dy)$$

$$= \int_{(0,1)}^{(3,3)} (2x^2 dx + 3y^2 dy)$$

Because the first integrand only involves x and the second integrand only involves y, W can be written as

$$W = \int_0^3 2x^2 dx + \int_1^3 3y^2 dy$$

Examining this last expression, we can directly conclude that the work done by the given force depends only on the endpoints A and D, and not on the particular path between the endpoints. Performing the integrations, we find

$$W = \frac{2x^3}{3}\Big|_{x=0}^{x=3} + y^3\Big|_{y=1}^{y=3}$$

$$= \frac{2}{3}[(3)^3 - (0)^3] + [(3)^3 - (1)^3]$$

$$= 18 + 26 = \underline{44\ J}$$

It is of course possible to explicitly obtain this value for each of the specific paths shown in the figure. We do not give the complete details but we indicate procedures which can be used for the three paths shown.

(a) Along the straight line AD, y = (2x/3) + 1; this can be used to write W as an integral involving x only.

(b) Along the path ABD, W can be written as the sum of an integral along AB (for which dx = 0) and an integral along BD (for which dy = 0).

(c) Along the path ACD, W can be written as the sum of an integral along AC (for which dx = 0) and an integral along the circular quadrant CD. The latter integral can be expressed in terms of the single variable θ, which is shown in the figure above. The quadrant extends from θ = 0 to θ = π/2. Representing the radius by R (it has the value 3 m), the coordinates along the arc are given by x = R sinθ and y = R(1 - cosθ). These expressions can be used to evaluate the integral $\int_C^D \vec{F}_1 \cdot d\vec{s}$.

7-28

(a) We denote the crate's mass and acceleration by m and a, respectively. After sliding a distance s from rest, the crate has a kinetic energy K given by $K \equiv \frac{1}{2}mv^2 = \frac{1}{2}m(2as) = mas$. With m = 50 kg, a = 2.0 m/s^2, and s = 10 m, we obtain K = 1000 J.

(b) The work W_g done on the crate by gravity is given by $W_g = mgh$, where h is the vertical distance descended by the crate. For an incline of angle θ, h = s sinθ, so we find W_g = mgs sinθ. With

$\theta = 30^{\circ}$ and the other given values, we find $W_g = 2450$ J. The only other force which does work on the crate is friction. If we let W_{fr} denote the work done by friction, we have $W_g + W_{fr} = K$. Therefore $W_{fr} = K - W_g = 1000 - 2450 = -1450$ J. The work spent in overcoming friction is $|-1450 \text{ J}| = \underline{1450 \text{ J}}$.

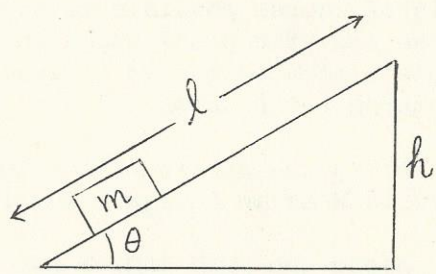
(c) The work W_{fr} done by friction is given by $W_{fr} = -F_{fr}s$, so that $F_{fr} = -W_{fr}/s = -(-1450)/(10) = \underline{145 \text{ N}}$.

(d) Since the crate remains in contact with the incline, the normal force $N = mg\cos\theta$. Then the frictional force $F_{fr} = \mu_k N = \mu_k mg\cos\theta$. Solving for μ_k, we obtain $\mu_k = F_{fr}/(mg\cos\theta) = 145/[(50)(9.8)(0.866)] = \underline{0.342}$.

(e) On a horizontal surface, the frictional force will be $F'_{fr} = \mu_k mg$. The crate will slide a distance s' such that the work W_{fr} done by friction equals the negative of the kinetic energy K. That is, $-\mu_k mgs' = -K$, so that $s' = K/(\mu_k mg) = 1000/[(0.342)(50)(9.8)] = \underline{5.97 \text{ m}}$.

7-29

(a) Referring to the figure at right, the increase in gravitational potential energy is given by $U_f - U_i = mgh = mg\ell\sin\theta$. With $m = 1.00$ kg, $\ell = 1.00$ m, and $\theta = 30^{\circ}$, we find $U_f - U_i = (1.00)(9.80)(1.00)(0.500) = \underline{4.90 \text{ J}}$.

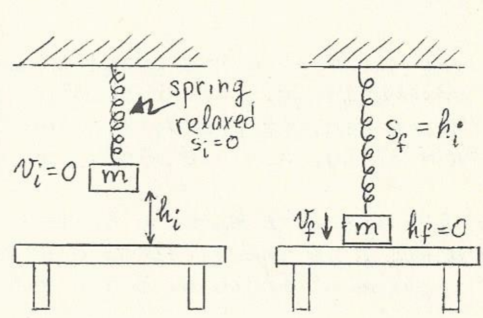


(b) The work $|W_{fr}|$ done against friction is $|F_{fr}\ell| = \mu N\ell = \mu(mg\cos\theta)\ell$. With $\mu = 0.30$, we find that $|W_{fr}|$ is equal to $(0.30)(1.00)(9.80)(8.66)(1.00) = \underline{2.55 \text{ J}}$.

(c) When the body slides down the ramp, the earth's gravity will do work W_g equal to the decrease in gravitational potential energy, while the frictional force will do work $-|F_{fr}\ell|$. If the block slides down from rest, its kinetic energy at the bottom will be $4.90 - 2.55 = \underline{2.35 \text{ J}}$.

7-30

The figures at the right show the system just as it is released, and just as the body strikes the table. The potential energy of the system may be written as $U = mgh + \frac{1}{2}ks^2$; h is the height above the table, and s is the amount of stretching



of the spring. When the object is released, $h = h_i = 0.15$ m $s = s_i = 0$, and the speed $v = v_i = 0$. When the object strikes the table, $h = h_f = 0$, $s = h_i = 0.15$ m, and $v = v_f$. The conservation of energy requires that

$$K_f + U_f = K_i + U_i \qquad \text{or}$$

$$\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}kh_i^2 = 0 + mgh_i$$

Solving for v_f, we find

$$v_f = \sqrt{2gh_i - (kh_i^2/m)}$$

With $k = 10$ N/m and $m = 0.100$ kg, we find that $v_f = \sqrt{2(9.80)(0.15) - [(10)(0.15)^2/(0.1)]} = \underline{0.831 \text{ m/s}}$.

7-31

(a) Referring to Fig. 7E-31, the conservation of energy yields the equation $\tfrac{1}{2}mv_B^2 + mgh_B = mgh_A$, so that $v_B = \sqrt{2g(h_B - h_A)}$. With $h_B - h_A = 1.0$ m, we obtain $v_B = \sqrt{2(9.8)(1.0)} = \underline{4.43 \text{ m}}$.

(b) If the block slides distance d in coming to rest, the work-energy theorem gives $W_{fr} = -\mu_k mgd = -\tfrac{1}{2}mv_B^2$. Then $\mu_k = (\tfrac{1}{2}mv_B^2)/(mgd) = mg(h_B - h_A)/(mgd) = (h_B - h_A)/d$. With $d = 3.0$ m, $\mu_k = 1/3 = \underline{0.333}$.

7-32

(a) We use x to denote the position of the free end of the spring, with $x = 0$ corresponding to the spring's relaxed position, and $x > 0$ corresponding to stretching. Initially the spring is compressed ($x_o = -0.100$ m), and the ball is at rest ($v_o = 0$). The spring loses contact with the ball, and the ball reaches its final speed v_f, just as the spring would begin to pull rather than push. That is, $x_f = 0$. The conservation of energy requires that

$$\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}kx_f^2 = \tfrac{1}{2}mv_o^2 + \tfrac{1}{2}kx_o^2$$

With $v_o = 0$ and $x_f = 0$, we find

$$v_o = |x_o|\sqrt{\frac{k}{m}}$$

With $k = 600$ N/m and $m = 0.200$ kg, we find $v_o = (0.10)\sqrt{(600)/(0.200)} = \underline{5.48 \text{ m/s}}$.

(b) Once again, the ball will lose contact with the spring just as the spring reaches its relaxed length. Measuring y positive upward from the relaxed position, the conservation of energy requires that

$$\frac{1}{2}mv_f^2 + \frac{1}{2}ky_f^2 + mgy_f = \frac{1}{2}mv_o^2 + \frac{1}{2}ky_o^2 + mgy_o.$$

Using the values $v_o = 0$, and $y_f = 0$, we obtain

$$v_f = \sqrt{\frac{ky_o^2}{m} + 2gy_o}$$

With $y_o = -0.100$ m, we find

$$v_f = \sqrt{[(600)(-0.1)^2/(0.2)] + (2)(9.80)(-0.1)}$$

$$= \underline{5.30 \text{ m/s}}$$

7-33

Referring to Fig. 7E-33, we use v_A and v_B to denote the speeds of the car at points A and B, and we use h_A and h_B to denote the elevations. If the car has mass m, the equation expressing energy conservation is

$$\frac{1}{2}mv_B^2 + mgh_B = \frac{1}{2}mv_A^2 + mgh_A$$

Since $v_A = 0$, we find $v_B^2 = 2g(h_A - h_B)$. The required centripetal force at B is mv_B^2/R, where R is the radius of curvature of the track at B. In order for the normal force exerted by the track to be positive, the centripetal force must be smaller in magnitude that the car's weight mg. That is, we must have

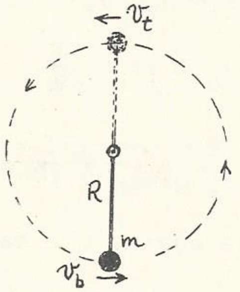
$$\frac{mv_B^2}{R} \leq mg$$

Solving for R, we find

$$R \geq R_{min} = v_B^2/g = 2(h_A - h_B)$$

With $h_A = 15.0$ m and $h_B = 5.0$ m, we obtain $R \geq 2(10) = \underline{20.0 \text{ m}}$.

7-34



The situation is shown in the figure at the right.

(a) As the bob passes through the bottom of the loop, it must be acted upon by a net upward force of magnitude mv_o^2/R. Allowing for the downward pull of gravity, the tension T_b satisfies the equation $T_b - mg = mv_o^2/R$, so that $T_b = mg + mv_o^2/R$.

(b) The top of the circular path is a distance 2R above the bottom, so the equation of energy conservation gives $\frac{1}{2}mv_t^2 + mg(2R) = \frac{1}{2}mv_o^2$, where v_t is the bob's speed at the top. Solving for v_t, we find $v_t = \sqrt{v_o^2 - 4gR}$.

(c) As the bob passes through the top of its path, it must be acted upon by a net downward force of magnitude mv_t^2/R. Here the tension T_t and the weight act in the same direction, so the force equation reads $T_t + mg = mv_t^2/R$. Solving for the tension we find $T_t = mv_t^2/R - mg = mv_o^2/R - 5\ mg$.

(d) The difference in tensions is given by $T_b - T_t = (mg + mv_o^2/R) - (mv_o^2/R - 5\ mg) = 6\ mg$.

(e) If $v_o^2 < 5gR$, we find that $T_t < 0$, which means that the rod is under compression.

(f) Since $v_t^2 = v_o^2 - 4gR$, the fact that the bob is executing complete circles indicates that $v_o > 2\sqrt{gR}$.

7-35

(a) In order for the car to remain in contact with the track at the top of the loop, the required downward centripetal force must equal or exceed the weight. Letting v_t denote the speed at the top, we have $mv_t^2/R \geq mg$, or $v_t \geq \sqrt{gR}$.

(b) Assuming that the car starts from rest, the kinetic energy at the top of the loop is given by $\frac{1}{2}mv_t^2 = mgh$. Solving for h, we obtain $h = v_t^2/2g$. Since $v_t \geq \sqrt{gR}$, we find $h \geq h_{min} = R/2$.

(c) Let h'_{min} be the minimum height needed when frictional forces are taken into account. Ideally the kinetic energy would be $K_{ideal} = mgh'_{min}$, but actually the kinetic energy is only mgh_{min}. The ratio $(K_{actual}/K_{ideal}) = h_{min}/h'_{min}$. Given that $h_{min} = 1.3\ h_{min}$, we find $K_{actual}/K_{ideal} = 0.769$.

7-36

(a) Referring to Fig. 7E-36, since the total length of the string does not change, the block A must move twice as far as the block B. That is, $s_A = 2s_B$, or $s_B = s_A/2$.

(b) Since $s_A = 2s_B$, $ds_A/dt = 2(ds_B/dt)$. Therefore $v_A = 2v_B$, or $v_B = v_A/2$.

(c) When body B moves a distance s_B (with s_B positive downward), the change in gravitational potential energy is $\Delta U = -m_B g s_B$. By conservation of energy, the change in kinetic energy $\Delta K = -\Delta U = m_B g s_B$. Since the system is released from rest, the speeds v_A and $v_B = v_A/2$ satisfy the equation

$$\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = m_B g D$$

when body B has descended a distance D. Substituting for v_B and solving for v_A, we find

$$v_A = \sqrt{\frac{8 m_B g D}{m_B + 4 m_A}} = \sqrt{\frac{8gD}{(1 + 4m_A/m_B)}}$$

(d) With $m_A = m_B$ and D = 2.0 m, the above equation yields $v_A = \sqrt{8(9.8)(2.0)/5} = \underline{5.60 \text{ m/s}}$.

<u>7-37</u>

(a) We denote the block's position by x (with $x > 0$ for a stretched spring). Initially $x = x_o = 0$ and $v \equiv dx/dt = v_o$. The initial total energy of the system is therefore $\frac{1}{2} m v_o^2$. When the rightward motion is arrested, $x = x_M$ and $v = 0$, so that the total mechanical energy is $\frac{1}{2} k x_M^2$. During the rightward motion there is a decrease in the total mechanical energy due to the (negative) work done by the frictional force. Therefore we have

$$\frac{1}{2} k x_M^2 = \frac{1}{2} m v_o^2 - \mu_k m g |x_M - x_o|$$

$$= \frac{1}{2} m v_o^2 - \mu_k m g x_M$$

Solving the quadratic equation for x_M and rejecting the negative root, we find

$$x_M = \sqrt{\left(\frac{\mu_k mg}{k}\right)^2 + \frac{m v_o^2}{k}} - \frac{\mu_k mg}{k}$$

(b) The maximum static frictional force is $\mu_s mg$. If that maximum force equals or exceeds the leftward restoring force $k x_M$, the block remains at $x = x_M$. That is, if $k x_M \leq \mu_s mg$, the block remains at $x = x_M$. If $k x_M > \mu_s mg$, the block starts to slide back.

(c) For m = 10 kg, k = 100 N/m, $\mu_s = 0.30$, $\mu_k = 0.15$, and $v_o = 1.0$ m/s, we find

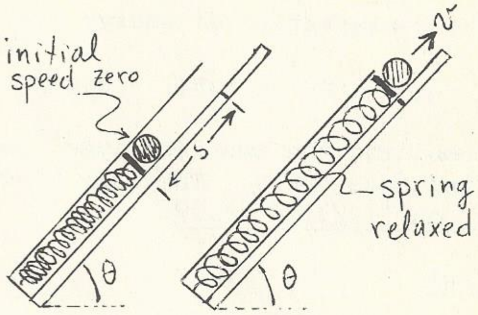
$$x_M = \sqrt{\left[\frac{(0.15)(10)(9.8)}{(100)}\right]^2 + \frac{(10)(1.0)^2}{(100)}} - \frac{(0.15)(10)(9.8)}{(100)}$$

$$= 0.3487 - 0.1470 = \underline{0.202 \text{ m}}$$

The restoring force $kx_M = 20.2$ N, while the maximum available static frictional force $\mu_s mg = 29.4$ N, so the block remains at $x = x_M$.

7-38

The initial and final situations are shown at right. We measure the elevation from the initial position of the mass. Initially, the energy is $\frac{1}{2}ks^2$, the compressional energy stored in the spring. The ball attains launch speed v when the spring ceases to push. When this occurs the elevation h above the initial position is given by $h = s \sin\theta$. The conservation of energy yields



$$\frac{1}{2}mv^2 + 0 + mgh = 0 + \frac{1}{2}ks^2 + 0$$

This gives $v^2 = ks^2/m - 2gs \sin\theta$, as desired.

7-39

As m descends a distance x_M to its momentary rest position (turning point), the total energy of the mass plus spring system is conserved. Since the initial and final kinetic energies are both zero, we have

$$0 = \frac{1}{2}kx_M^2 - mgx_M$$

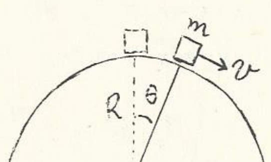
so

$$x_{max} = \frac{2mg}{k}$$

For a spring balance calibrated to display weights, this results in a momentary scale reading equal to 2mg, which is twice the actual weight of m.

7-40

Referring to the figure at right, the radial component of the force equation for the particle is $N - mg \cos\theta = -mv^2/R$. Here N is the normal force exerted by the dome. Since the particle starts from rest at $\theta = 0$, the normal force is initially equal to the weight mg, but N decreases as θ



increases. The particle leaves the dome surface when the equation above first predicts a negative normal force. This occurs for $mv^2/R = mg\cos\theta$, or when

$$\frac{1}{2}mv^2 = \frac{1}{2}mgR\cos\theta$$

The conservation of energy yields

$$\frac{1}{2}mv^2 = mgR(1 - \cos\theta)$$

Combining the two displayed equations and solving for $\cos\theta$, we find $\cos\theta = 2/3$. The body leaves the dome surface at $\underline{\theta = \cos^{-1}(2/3) = 48.2^o}$.

7-41

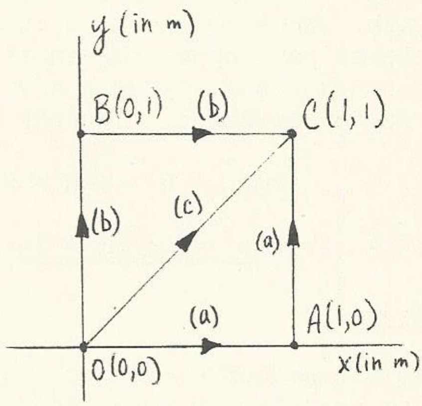


Figure 7E-41 is reproduced at the right. For each of the paths from O to C, the work done by F_2 is given by $W = \int_0^C \vec{F}_2 \cdot d\vec{s}$, where $\vec{F}_2 = xy\hat{x} + xy\hat{y}$ and $d\vec{s} = \hat{x}dx + \hat{y}dy$. The dot product $\vec{F}_2 \cdot d\vec{s} = xydx + xydy$.

(a) The path OAC consists of the straight lines OA and AC. Along OA, y = 0 and dy = 0, so $\int_{CA} \vec{F}_2 \cdot d\vec{s} = 0$. Along AC, x = 1 m and dx = 0, so $\vec{F}_2 \cdot d\vec{s} = ydy$. Therefore

$$\int_{AC} \vec{F}_2 \cdot d\vec{s} = \int_0^1 ydy = \frac{y^2}{2}\Big|_0^1 = \frac{1}{2}\ \text{J}$$

The work done along the entire path OAC is given by

$$W_{OAC} = \int_{OA} \vec{F}_2 \cdot d\vec{s} + \int_{AC} \vec{F}_2 \cdot d\vec{s} = 0 + \frac{1}{2}\ \text{J} = \underline{\frac{1}{2}\ \text{J}}$$

(b) The path OBC consists of the straight lines OB and BC. Along OB, x = 0 and dx = 0, so $\int_{OB} \vec{F}_2 \cdot d\vec{s} = 0$. Along BC, y = 1 m and dy = 0, so $\vec{F}_2 \cdot d\vec{s} = xdx$. Therefore

$$\int_{BC} \vec{F}_2 \cdot d\vec{s} = \int_0^1 xdx = \frac{x^2}{2}\Big|_0^1 = \frac{1}{2}\ \text{J}$$

The work done along the entire path OBC is given by

$$W_{OBC} = \int_{OB} \vec{F}_2 \cdot d\vec{s} + \int_{BC} \vec{F}_2 \cdot d\vec{s} = 0 + \frac{1}{2}\,J = \frac{1}{2}\,J$$

(c) Along the straight line OC, $y = x$, so $dy = dx$ and $\vec{F}_2 \cdot d\vec{s} = xydx + xydy = 2x^2dx$. Then

$$W_{OC} = \int_{OC} \vec{F}_2 \cdot d\vec{s} = \int_0^1 2x^2 dx = \left.\frac{2x^3}{3}\right|_0^1 = \frac{2}{3}\,J$$

(d) The force $\vec{F}_2$ is <u>not</u> conservative because the work done by $\vec{F}_2$ between two points depends upon the particular path between the two points. In contrast to this, the force $\vec{F}_1$ of Exercise 7-27 is conservative. The fact that $\int \vec{F}_1 \cdot d\vec{s}$ can be written in a form that is obviously path-independent establishes the conservative nature of the force $\vec{F}_1$.

7-42

(a) With $dM/dt = -\lambda$, the mass $M(t) = M_i - \lambda t$. The angular frequency $\omega(t) = \sqrt{k/M(t)} = \sqrt{k/(M_i - \lambda t)}$.

(b) The instantaneous rate of loss of kinetic energy is given by $-dK/dt = \frac{1}{2}(-dM/dt)v^2(t) = \frac{1}{2}\lambda v^2(t)$. The loss of kinetic energy in one cycle is $-\Delta K = \overline{(-dK/dt)} \cdot 2\pi/\omega(t) = (\pi\lambda/\omega)\overline{v^2}$. Because v varies sinusoidally, $\overline{v^2} = \frac{1}{2}v_{max}^2 = \frac{1}{2}\omega^2(t)A^2(t)$. The loss per cycle $-\Delta K$ is therefore given by $-\Delta K = \pi\lambda\omega A^2/2$.

(c) The rate of loss of mechanical energy is $-\overline{(dK/dt)} = -(\omega\Delta K/2\pi) = \lambda \cdot \omega^2 A^2/4$. Since the total energy of a harmonic oscillator is $\frac{1}{2}kA^2(t)$, we obtain the equation

$$\frac{d}{dt}\left[\frac{1}{2}kA^2(t)\right] = \frac{-\lambda}{4}\omega^2(t)A^2(t)$$

Since $\omega^2(t) = k/M(t)$ and $-\lambda = dM/dt$, the displayed equation implies that

$$\frac{1}{A^2}\frac{d(A^2)}{dt} = \frac{1}{2}\left(\frac{1}{M}\frac{dM}{dt}\right) \qquad \text{or}$$

$$\frac{1}{A}\frac{dA}{dt} = \frac{1}{4}\left(\frac{1}{M}\frac{dM}{dt}\right)$$

This can be integrated to give

$$\frac{A(t)}{A_i} = \left[\frac{M(t)}{M_i}\right]^{1/4} = [1 - (\lambda t/M_i)]^{1/4}$$

(d) The final oscillation frequency $\omega_f = \sqrt{k/M_f} = \omega_i(M_f/M_i)^{-1/2}$. The final amplitude $A_f = A_i(M_f/M_i)^{1/4}$. The final oscillation energy $E_f = E_i(M_f/M_i)^{1/2}$.

(e) For $M_f = 0.10\ M_i$, we find $\omega_f = \omega_i\sqrt{10} = \underline{3.16\ \omega_i}$, $A_f = A_i(0.1)^{1/4} = \underline{0.562A_i}$, and $E_f = E_i\sqrt{0.10} = \underline{0.316E_i}$.

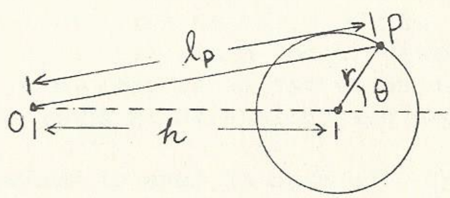
7-43

(a) The potential energy $U = \frac{1}{2}k(\ell - \ell_o)^2$. When the body is at point A, $\ell = h + r$, so $\underline{U_A = \frac{1}{2}k(h + r - \ell_o)^2}$.

(b) When the body is at point B, $\ell = |h - r|$, so $U_B = \frac{1}{2}k(|h - r| - \ell_o)^2$. If $h > r$, then $U_B = \frac{1}{2}k(h - r - \ell_o)^2$. If $h \leq r$, then $U_B = \frac{1}{2}k(r - h - \ell_o)^2$.

(c) The work $W_{A\to B}$ done by the spring as the body moves from point A to point B is given by $W_{A\to B} = U_A - U_B$. If $h > r$, then $W_{A\to B} = \frac{1}{2}k[(h + r - \ell_o)^2 - (h - r - \ell_o)^2] = 2kr(h - \ell_o)$. If $h \leq r$, then $W_{A\to B} = \frac{1}{2}k[(h + r - \ell_o)^2 - (r - h - \ell_o)^2] = 2kh(r - \ell_o)$.

(d) The potential energy $U_P = \frac{1}{2}k(\ell_P - \ell_o)^2$, where ℓ_P is the length of the spring. Applying the law of cosines to the figure at right, we have



$$\ell_P = \sqrt{h^2 + r^2 - 2hr\cos(\pi - \theta)} = \sqrt{h^2 + r^2 + 2hr\cos\theta}$$

Therefore

$$U_P = \frac{1}{2}k\left(\sqrt{h^2 + r^2 + 2hr\cos\theta} - \ell_o\right)^2$$

(e) (1) If $h + r < \ell_o$, then $h < \ell_o$ and the speed of the body decreases as it moves from A toward B. The body may or may not reach B. Letting $Q \equiv \frac{1}{2}mv_A^2 + (U_A - U_B)$, if $Q<0$, the body stops and reverses its motion. However, if $Q > 0$ the body goes past point B and then speeds up as it travels counterclockwise back to point A, completing a circular motion which it will repeat indefinitely. If $Q = 0$, the body stops at point B. (2) If $|h - r| < \ell_o < h + r$, then the body speeds up at first and then slows down as the spring begins to be compressed. As in (1), if $Q < 0$, the body stops and reverses its motion. If $Q > 0$, the body executes complete counterclockwise circles. If $Q = 0$, then the body stops at B. (3) If $|h - r| > \ell_o$, then $W_{A\to B} = U_A - U_B > 0$, so the body's speed increases all the way from A to B. The body executes complete counterclockwise circles. (4) If h = 0, then $U_p = \frac{1}{2}k(r - \ell_o)^2$ = constant, so the body executes counterclockwise circular motion at constant speed.

(f) Only the answers to parts (a), (b), and (d) would be affected. The answers to (c) and (e) are based upon potential energy differences, which are not altered by a change in the reference position.

7-44

(a) We take the potential energy to be zero when $\ell = \ell_o$. The initial separation $\ell_i = h + 2r$, so the initial potential energy $U_i = \frac{1}{2}k(h + 2r - \ell_o)^2$. Since the initial kinetic energy is zero, the conservation of total energy implies that

$$\frac{1}{2}mv_C^2 + \frac{1}{2}mv_C^2 + U_{CC'} = U_i$$

Here v_C is the speed of each body when the two objects are passing through points C and C'. The potential energy $U_{CC'} = \frac{1}{2}k(h - 2r - \ell_o)^2$. In order for the bodies to reach points C and C', we must have $(h - 2r - \ell_o)^2 \leq (h + 2r - \ell_o)^2$. This requires $-4r(h - \ell_o) \leq 4r(h - \ell_o)$, or $\underline{h \geq \ell_o}$.

(b) Using the equation displayed above, we obtain

$$v_C^2 = \frac{1}{m}\left(U_i - U_{CC'}\right)$$

$$= \frac{k}{2m}\left[(h + 2r - \ell_o)^2 - (h - 2r - \ell_o)^2\right]$$

$$= \frac{k}{2m}\left[(4r)(h - \ell_o) - (-4r)(h - \ell_o)\right]$$

$$= \frac{4kr}{m}(h - \ell_o)$$

Each body must be acted upon by a net force $mv_C^2/r = 4k(h - \ell_o)$, directed toward the center of its track. The force supplied by the spring is $-k(h - 2r - \ell_o)$, so the track must supply a contact force F_T such that $F_T - k(h - 2r - \ell_o) = 4k(h - \ell_o)$. Solving for F_T, we find $\underline{F_T = 5k(h - \ell_o) - 2kr}$. (NOTE: $F_T > 0 \Leftrightarrow \vec{F}_T$ radially inward.)

(c) In order that $F_T = 5k(h - \ell_o) - 2kr = 0$, we must have $5(h - \ell_o) = 2r$, or $\underline{h = \ell_o + 2r/5}$.

7-45

We denote the spring constant, the relaxed length, and the collar mass by k, ℓ_o, and m, respectively. We let v_i and ℓ_i represent the initial collar speed and spring length; v and ℓ represent the instantaneous speed and spring length at some other time. The conservation of mechanical energy implies that

$$\frac{1}{2}mv^2 + \frac{1}{2}k(\ell - \ell_o)^2 = \frac{1}{2}mv_i^2 + \frac{1}{2}k(\ell_i - \ell_o)^2$$

Since $v_i = 0$, we have

$$v = \sqrt{\frac{k}{m}}\left[(\ell_i - \ell_o)^2 - (\ell - \ell_o)^2\right]^{1/2}$$

Referring to Figure 7E-45,

$$\ell_i = \sqrt{(OA)^2 + (AS)^2} = \sqrt{(0.20)^2 + (0.15)^2} = 0.25 \text{ m}$$

(a) As the collar passes point A, $\ell = \ell_A = 0.20$ m. With $\ell_o = 0.10$ m, $k = 500$ N/m, and $m = 10$ kg, we obtain

$$v_A = \sqrt{\frac{500}{10}}\left[(0.25 - 0.10)^2 - (0.20 - 0.10)^2\right]^{1/2}$$

$$= \sqrt{50}\sqrt{0.0225 - 0.0100}$$

$$= \underline{0.791 \text{ m/s}}$$

(b) As the collar passes point B, $\ell = \ell_B = \sqrt{(OA)^2 + (AB)^2}$ $= \sqrt{(0.20)^2 + (0.10)^2} = 0.2236$ m. Then we have

$$v_B = \sqrt{50}\left[(0.15)^2 - (0.1236)^2\right]^{1/2}$$

$$= \underline{0.601 \text{ m/s}}$$

CHAPTER EIGHT

8-11

The pulley systems of Figure 8E-11 are reproduced on the next page, with notations indicated for various pulleys, ropes, and elevations. In each case, we let h_m denote the height (above the floor) of the body of mass M, we let h_f denote the height (above the floor) of the point of application of the force F, and we let s denote the total length of rope which has been pulled past the point of application of the force. Then, by conservation of energy, the mechanical advantage Mg/F must be equal to $\Delta s/\Delta h_m$. In each case, we assume that the point of application of the force maintains a constant elevation as the pulleys are moved: $\Delta h_f = 0$.