In order for this quantity to be positive, we must have

$$\frac{3}{8} > \frac{r}{R + r}$$

Solving this for R, we find that for stability we need $R > R_{min} = \underline{5r/3}$.

CHAPTER TEN

10-6

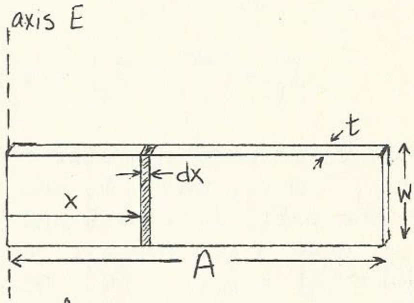
As shown in Example 10-6 and summarized there in Equation 10-36, the precessional angular velocity ω_p is given by

$$\omega_p = \frac{rg}{G^2}\frac{1}{\omega_s}$$

where ω_s is the spin angular velocity, G is the radius of gyration of the object, r is the distance from the point of support to the center of mass, and g is the acceleration due to gravity. In the present exercise, the effective gravity increases from g to g' = 4g as the space bus achieves an upward acceleration of 3g. Since the quantities r, G, and ω_s are unchanged, the precessional angular velocity must increase from ω_p to $\underline{\omega_p' = 4\omega_p}$.

10-7

It should be noted that the thickness t shown in Figure 10-7 is assumed to be negligible (as in Example 10-1): $t \ll A$.



(1) First we find I_E by direct integration. Referring to the figure at the right, we have $dM = (M/A)dx$. Therefore

$$I_E = \int x^2 dM = \int_0^A (M/A)x^2 dx = \left.\frac{Mx^3}{3A}\right|_0^A = \frac{1}{3}MA^2$$

(2) The second method employs the parallel-axis theorem. The moment of inertia I_c about an axis through the center of the rod was shown in Example 10-1 to be given by $I_c = MA^2/12$. The parallel-axis theorem implies that

$$I_E = I_c + M(A/2)^2 = \frac{MA^2}{12} + \frac{MA^2}{4} = \frac{MA^2}{3}$$

in agreement with the result obtained by direct integration.

10-8

For the equation $\vec{T} = I\vec{\alpha}$ to be valid, the origin must be at the center of mass of the object or else in the plane of rotation at a point which is fixed in an inertial frame of reference. The shaft goes through the geometrical center of the wheel, which is (a) in the plane of rotation and (b) fixed in an inertial frame. We therefore adopt the geometrical center of the wheel as the origin. The moment of inertia I' of the chipped wheel (about the shaft) is

$$I' = \frac{1}{2} M_o R^2 - R^2(\Delta M)$$

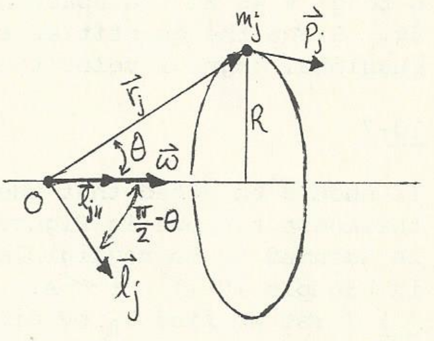
$$= [(0.5)(100) - 4](0.32)^2$$

$$= 4.71 \text{ kg m}^2$$

The angular acceleration must have magnitude $|\vec{\alpha}| = |\Delta\vec{\omega}|/\Delta t = (200 \times 2\pi)/(3 \times 60) = 6.98 \text{ rad/s}^2$. The required torque has magnitude $I'|\vec{\alpha}|$ = 32.9 N-m.

10-9

The figure is shown at right. The magnitude ℓ_j of the angular momentum $\vec{\ell}_j$ of the particle having position $\vec{r}_j$ and momentum $\vec{p}_j$ is



$$\ell_j = |\vec{r}_j \times \vec{p}_j| = r_j p_j$$

since $\vec{r}_j$ is perpendicular to $\vec{p}_j$. Now $p_j = m_j v_j$, where m_j and v_j are the particle's mass and speed, so we have $\ell_j = r_j m_j v_j$. But $v_j = |\vec{\omega} \times \vec{r}_j| = \omega r_j \sin\theta$. Here θ is the angle between $\vec{\omega}$ and $\vec{r}_j$. As can be seen from the figure, the angle between $\vec{\ell}_j$ and $\vec{\omega}$ is $(\pi/2 - \theta)$. The component $\ell_{j\parallel}$ of $\vec{\ell}_j$ along $\vec{\omega}$ is therefore given by

$$\ell_{j\parallel} = \ell_j \cos(\pi/2 - \theta)$$

$$= m_j r_j^2 \omega \sin^2\theta$$

Since by symmetry the perpendicular vectors $\vec{\ell}_{j\perp}$ add to zero, we have

$$\vec{L} = \hat{\omega} L_{\parallel} = \hat{\omega} \sum_{j=1}^{n} \ell_{j\parallel} = \vec{\omega} \sum_{j=1}^{n} m_j r_j^2 \sin^2\theta$$

But the figure shows that $r_j \sin\theta = R$ for all n particles.

$\vec{L} = \vec{\omega} R^2 \sum_{j=1}^{n} m_j = MR^2\vec{\omega}$, where $M \equiv \sum_{j=1}^{n} m_j$ is the total mass of the circling particles.

10-10

(a) Referring to Figure 10E-10, since $x^2 + y^2 = R^2$, we have $x^2 dm + y^2 dm = R^2 dm$, or $\underline{dI_y + dI_x = dI_z}$.

(b) The relationship just obtained holds for any mass element on the ring. Therefore we may integrate the expression over the ring. We find $\underline{I_x + I_y = I_z}$.

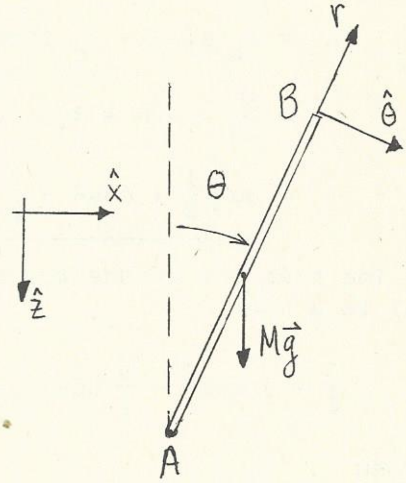
(c) Symmetry demands that the moment of inertia I_x about the x-axis equal the moment of inertia I_y about the y-axis. (See Figure 10E-10). That is, $\underline{I_y = I_x}$.

(d) It is easily seen that $I_z = \int R^2 dm = R^2 \int dm = \underline{MR^2}$. Since $I_z = I_x + I_y = 2I_x$, we find $I_x = I_y = \underline{MR^2/2}$.

(e) Referring to Figure 10E-10, we apply the parallel axis theorem to find that

$$I_{Ax} = I_x + M(R)^2 = \underline{3MR^2/2}$$

10-11



(a) Using the notation indicated in the figure at the right, and calculating torques about point A, with clockwise being positive, we have

$$I_A \alpha = \frac{Mg\ell}{2} \sin\theta$$

where $I_A = M\ell^2/3$, as shown in Example 10-5 and in Exercise 10-7. Therefore the angular acceleration is given by

$$\alpha = \frac{3g}{2\ell} \sin\theta$$

(b) Recognizing that the force at the pivot does no work and using the conservation of energy, we find that

$$\frac{1}{2} I_A \omega^2 = \frac{Mg\ell}{2} (1 - \cos\theta)$$

Solving for ω^2, we find

$$\omega^2 = \frac{3g}{\ell} (1 - \cos\theta)$$

Using the unit vectors $\hat{\theta}$ and $\hat{r}$ shown in the figure, the linear acceleration a of a point on the rod a distance r from the pivot is given by

$$\vec{a} = a_t\hat{\theta} + a_c\hat{r}$$

where the transverse acceleration $a_t = r\alpha$ and the centripetal acceleration $a_c = -\omega^2 r$. Using the expressions already obtained for α and ω, and setting $r = \ell$, we find that the free end of the rod has an acceleration

$$\vec{a} = \left(\frac{3g \sin\theta}{2}\right) \hat{\theta} + \left[- 3g(1 - \cos\theta)\right] \hat{r}$$

The magnitude of this vector is

$$a = 3g\sqrt{\frac{5}{4} - 2 \cos\theta + \frac{3}{4} \cos^2\theta}$$

(c) Using the unit vectors $\hat{x}$ and $\hat{z}$ shown in the figure, we have $\vec{a} = a_x\hat{x} + a_z\hat{z}$, where

$$a_z = a_t \sin\theta - a_c \cos\theta$$

$$= \frac{3g}{2} \sin^2\theta + 3g(1 - \cos\theta) \cos\theta$$

$$= 3g \left[\frac{1}{2} + \cos\theta - \frac{3}{2} \cos^2\theta\right]$$

(d) The free end of the rod has a downward acceleration component equal to g when

$$\frac{3}{2} + 3 \cos\theta - \frac{9}{2} \cos^2\theta = 1$$

or when

$$9 \cos^2\theta - 6 \cos\theta - 1 = 0$$

Solving the quadratic equation, we find $\cos\theta = \frac{1 \pm \sqrt{2}}{3}$, so that the first two values of θ at which $a_z = g$ are:

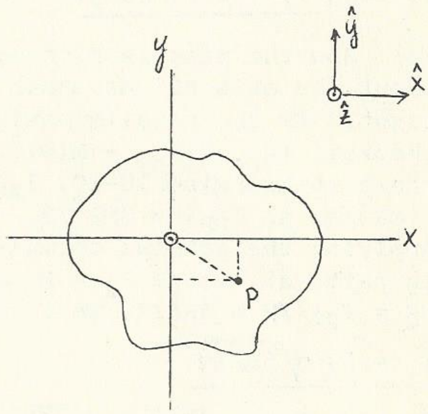
$$\theta_1 = \cos^{-1}\left(\frac{1 + \sqrt{2}}{3}\right) = \cos^{-1}(0.8047) = \underline{36.4^\circ}$$

and

$$\theta_2 = \cos^{-1}\left(\frac{1 - \sqrt{2}}{3}\right) = \cos^{-1}(-0.1381) = \underline{97.9^\circ}$$

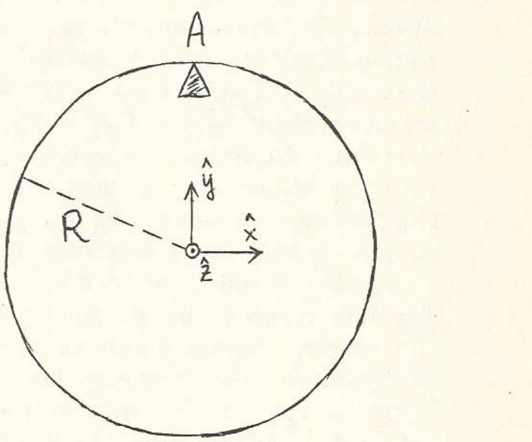
Naturally, there are two additional angles ($360^\circ - \theta_2 = 262.1^\circ$ and $360^\circ - \theta_1 = 323.6^\circ$) at which $a_z = g$.

10-12



The figure at the right indicates the general situation. The positive z-axis rises directly toward the viewer. An infinitesimal mass dm located at the arbitrary point P contributes an amount $dI_x = y^2 dm$ to the body's moment of inertia about the x-axis, and an amount $dI_y = x^2 dm$ to the moment of inertia about the y-axis. It also contributes an amount $dI_z = R^2 dm$ to the moment of inertia about the z-axis. Since $R^2 = y^2 + x^2$, we have $dI_z = dI_x + dI_y$. Integrating this equation over the entire object, we obtain $\underline{I_z = I_x + I_y}$. This result is known as the perpendicular-axis theorem.

10-13



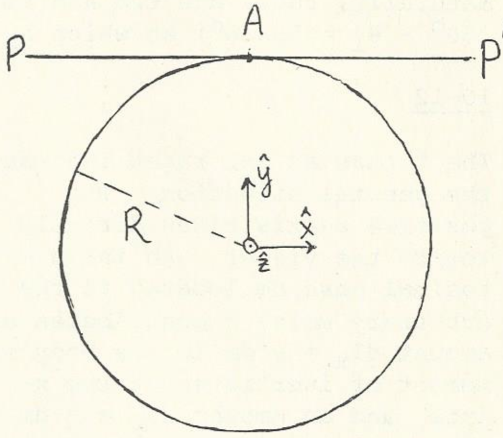
(a) The ring is shown at right, with the knife edge at point A. We must find the period T_1 of small oscillations in the plane of the paper. Taking the origin of a coordinate system at O, the equilibrium position of the ring's center, with the positive z-axis emerging toward the viewer, the moment of inertia $I_{zO} = MR^2$. By the parallel-axis theorem, the moment of inertia about the knife-edge is given by $I_{zA} = I_{zO} + M(R)^2 = 2MR^2$. By Equation 10-23, frequency ν_1 of small oscillations is given by

$$\nu_1 = \frac{1}{2\pi}\sqrt{\frac{D_1 g}{G_1^2}}$$

where G_1 is the radius of gyration about the axis of the pivot and D_1 is the distance between the pivot axis and a parallel axis through the center of mass. Therefore the period is

$$T_1 = 2\pi\sqrt{\frac{G_1^2}{D_1 g}}$$

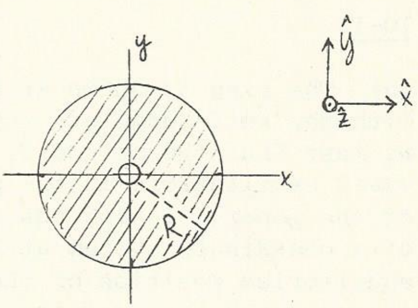
With $G_1^2 = I_{zA}/M = 2R^2$ and $D_1 = R$, we find $T_1 = 2\pi\sqrt{2R/g}$.



(b) Now the ring is pivoted about the axis PP' as shown at right. By the parallel-axis theorem, $I_{PP'} = I_{xO} + M(R)^2$. As shown in Exercise 10-10, $I_{xO} = MR^2/2$, so $I_{PP'} = 3MR^2/2$. Applying the general formula as in part (a), using $D_2 = R$ and $G_2^2 = I_{PP'}/M = 3R^2/2$, we find $T_2 = 2\pi\sqrt{3R/2g}$.

(c) $T_1/T_2 = \sqrt{4/3} = 2\sqrt{3}/3 = 1.1547$. The period for oscillations in the plane is 15.5% longer than the period for oscillations about the axis PP'.

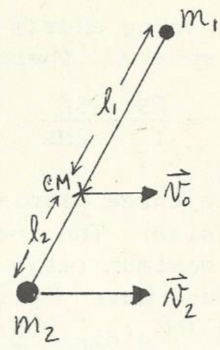
10-14



A thin uniform-density disk of radius R is confined to the xy plane, as shown at right. The perpendicular axis theorem implies that $I_{xO} + I_{yO} = I_{zO}$. But symmetry implies that $I_{xO} = I_{yO} = I_d$, where I_d denotes the moment of inertia about any diameter. Therefore, we have $2I_d = I_{zO}$. We now check this against Table 10-1. The moments of inertia of a solid cylinder of radius R and length A are as given: $\frac{1}{2}MR^2$ for the moment about the symmetry axis, and $(\frac{1}{4}MR^2 + \frac{1}{12}MA^2)$ for the moment about an axis through the center of mass and perpendicular to the symmetry axis. In the limit $A \to 0$, we have $\frac{1}{2}MR^2 \to I_{zO}$ and $(\frac{1}{4}MR^2 + \frac{1}{12}MA^2) \to \frac{1}{4}MR^2 \to I_d$. Thus the relationship $2I_d = I_{zO}$ for a thin disk is confirmed.

10-15

The situation just before the moment of release is shown in the figure at right. We observe that after release (1) the mutual forces (transmitted by the string) exerted by the bodies act through the center of mass and therefore exert no net torque on the system, and (2) there is no gravitational torque about the center of mass. Therefore, even though the center of mass of the system accelerates differently after release (than before release), the internal angular momentum of the system must be the same after release as before. By internal angular momentum, we mean angular momentum with respect to the (moving) center of mass. The speeds v_2 and v_o indicated in the figure are given by $v_2 = (\ell_1 + \ell_2)\omega_o$ and $v_o = \ell_1\omega_o$. The magnitude L_{int} of the internal angular momentum is given by



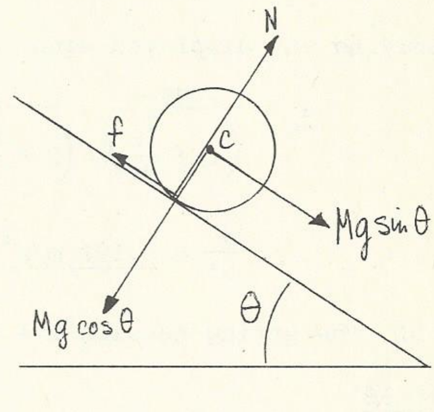
$$L_{int} = m_1\ell_1 v_o + m_2\ell_2(v_2 - v_o)$$

$$= m_1\ell_1^2\omega_o + m_2\ell_2^2\omega_o$$

After release, the internal angular momentum L_{int} will again be given by $L'_{int} = (m_1\ell_1^2 + m_2\ell_2^2)\omega'$. Angular momentum conservation ($L'_{int} = L_{int}$) therefore implies that $\omega' = \omega_o$. The angular velocity is unchanged: the angular speed after release is 2 rotations per second, or 4π rad/s.

10-16

The problem is diagrammed at right. We let α denote the angular acceleration (clockwise positive) and a denote the linear acceleration along the incline. If there is no slipping, then $v = \omega R$, where v and ω are the linear and angular velocities. Since $a = dv/dt$ and $\alpha = d\omega/dt$, the condition that the sphere does not slip can be written as



$$a = \alpha R$$

The translational acceleration is given by

$$a = g \sin\theta - f/M$$

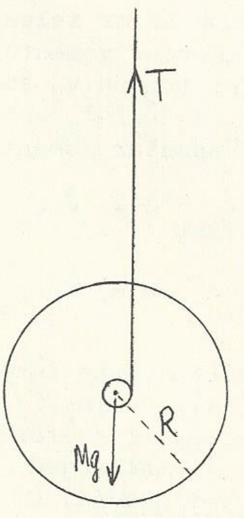
where f is the frictional force. Neither the sphere's weight nor the normal force exerts any torque about C, the center of the (uniform) sphere. Therefore the angular acceleration is given by

$$\alpha = \frac{fR}{I_C} = \frac{5f}{2MR}$$

Solving the three displayed equations for f, we find f = (2/7)Mg sinθ. The normal force N = Mg cosθ, so f/N = (2/7) tanθ. Since the maximum ratio (f_{max}/N) consistent with pure rolling is μ_s, we must have $\mu_s \geq (2/7)$ tanθ. For θ = 30°, tanθ = $1/\sqrt{3}$, so we find $(\mu_s)_{min}$ = $\underline{2/(7\sqrt{3}) = 0.165}$.

10-17

(a) The situation is shown at right. Taking counterclockwise rotations and downward translations as positive, we have the translational equation



$$Ma_C = Mg - T$$

the rotational equation

$$I_C\alpha = Tr$$

where $I_C = \frac{1}{2}MR^2$, and the constraint equation

$$a_C = \alpha r$$

Solving the displayed equations for a_C, we find

$$a_C = \frac{gr}{\left(r + \frac{I_C}{Mr}\right)} = \frac{g}{\left(1 + \frac{R^2}{2r^2}\right)}$$

$$= \frac{g}{51} = \underline{0.192 \text{ m/s}^2}$$

(b) The string tension T = M(g − a_C) = $\underline{50Mg/51 = 0.980Mg}$.

10-18

(a) Using the kinematic equation s = $\frac{1}{2}at^2$, we find a = $2s/t^2$ = 100 cm/s² and α = a/R = $\underline{5.00 \text{ rad/s}^2}$.

(b) The force F exerts a torque FR = Iα, so I = FR/α = (50 N)(0.2 m)/(5.0) = $\underline{2.00 \text{ kg-m}^2}$ = 2.00 x 10^7 g-cm².

(c) In this case the acceleration a' of the object is given by Ma' = W - F', where F' is the tension in the rope. Furthermore, F' R = Iα'. The linear and angular accelerations are related as before: a' = α'R. Solving for α', we find

$$\alpha' = \frac{W}{\left(\frac{WR}{g} + \frac{I}{R}\right)} = \frac{50}{\left[\frac{(50)(0.2)}{(9.80)} + \left(\frac{2.00}{0.20}\right)\right]}$$

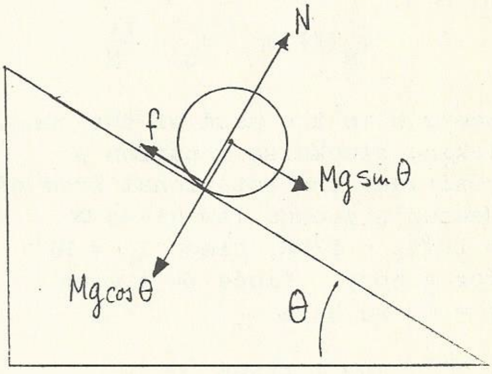
$$= \underline{4.54 \text{ rad/s}^2}$$

(d) The angular acceleration $\alpha' < \alpha$ because part of the weight force W is "expended" in accelerating the suspended mass. Thus F' = W - Ma' < W, so $\alpha' < \alpha$ (even though W = F).

(e) A uniform disk of radius R has a moment of inertia $I = \frac{1}{2}MR^2$ about the symmetry axis. Therefore $M = 2I/R^2 = 2(2.00)/(0.20)^2$ = $\underline{100 \text{ kg}}$ = 1.00×10^5 g.

10-19

The diagram at right indicates the general situation in each case. The acceleration a down the incline is determined by three equations. The first is Newton's second law,



$$Ma = Mg \sin\theta - f$$

where f is the frictional force. The second equation is the rotational form of Newton's second law,

$$I\alpha = fR$$

where I is the moment of inertia of the object about its symmetry axis. The third equation is the rolling constraint

$$a = \alpha R$$

Solving the three displayed equations for a, we find

$$a = \frac{g \sin\theta}{(1 + I/MR^2)}$$

(a) For a solid cylinder $I = I_c = \frac{1}{2}MR^2$, so $a_c = g \sin\theta/(1 + 1/2)$ = $\underline{(2g/3) \sin\theta}$.

(b) For a thin-walled pipe, $I = I_p = MR^2$, so $a_p = g \sin\theta/(1 + 1)$ $= \underline{(g/2)\ \sin\theta}$.

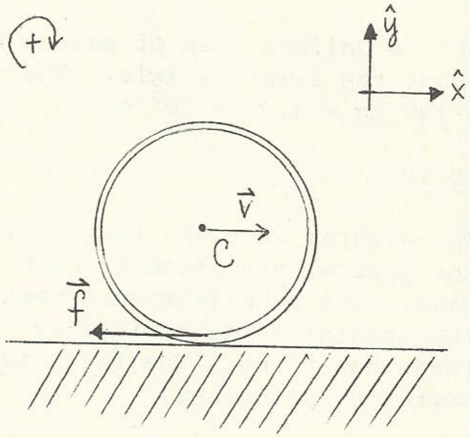
(c) The distances traveled are in the same ratio as the accelerations:

$$\frac{s_p}{s_c} = \frac{a_p}{a_c} = \frac{(1/2)}{(2/3)} = \frac{3}{4}$$

That is, $s_p = \underline{3\ s_c/4}$.

10-20

The situation is shown at right. We take $t = 0$ to denote the instant at which the hoop is projected with velocity $\vec{v} = v_o\hat{x}$. If the frictional force $\vec{f} = -f\hat{x}$, Newton's second law gives



$$v_x(t) = v_o - \frac{ft}{M}$$

where M is the mass of the hoop. Taking clockwise rotation as positive, the rotational form of Newton's second law gives $\alpha = fR/I_C = f/MR$, since $I_C = MR^2$ for a hoop. Since $\omega = 0$ at $t = 0$, we have

$$\omega = ft/MR$$

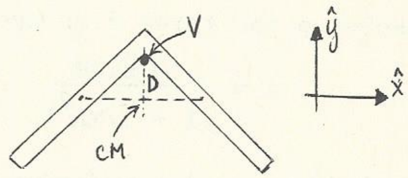
The hoop will stop sliding when $v_x = \omega R$, or when

$$v_x(t) = v_o - \frac{ft}{M} = \frac{ft}{M}$$

Therefore the hoop will slide until $t = Mv_o/2f$, at which time its velocity will be $v_o - (f/M)(Mv_o/2f) = \underline{v_o/2}$.

10-21

The equilibrium configuration is shown at the right. As established in Exercise 9-53, the center of mass of the system is located on the angle bisector, at a distance $D = (\ell\sqrt{2}/4)$ from the vertex V. The moment of inertia I about an axis through V and



perpendicular to the xy plane is given by $(m\ell^2/3) + (m\ell^2/3) = 2m\ell^2/3$. (See Exercise 10-7.) Therefore the square of the gyration radius is given by $G^2 \equiv I/M = (2m\ell^2/3)(2m) = \ell^2/3$. We can now apply Equation 10-23:

$$T \equiv \frac{1}{\nu} = 2\pi\sqrt{\frac{G^2}{Dg}}$$

$$= 2\pi\sqrt{\frac{(\ell^2/3)}{(\ell\sqrt{2}/4)g}} = 2\pi\sqrt{\frac{2\sqrt{2}\,\ell}{3g}}$$

as desired.

<u>10-22</u>

The parallel-axis theorem (Eq. 10-14) is not restricted to two-dimensional objects, nor to axes parallel to the one of the principal axes. Therefore the moment of inertia about axis PP' is given by

$$I_{PP'} = I_C(ZZ') + MD^2$$

where $I_C(ZZ')$ is the moment of inertia about axis ZZ'. Using the definition of the gyration radius, we find

$$I_{PP'} = M[G_C^2(ZZ') + D^2]$$

Furthermore, the formula for the frequency of small oscillations of a physical pendulum (Eq. 10-23) is not restricted to two-dimensional objects, nor to axes parallel to one of the principal axes. Therefore, the frequency of pendular oscillations about PP' is given by

$$\nu = \frac{1}{2\pi}\sqrt{\frac{Dg}{I_{PP'}/M}} = \frac{1}{2\pi}\sqrt{\frac{Dg}{G_C^2(ZZ') + D^2}}$$

as desired.

<u>10-23</u>

This exercise is a special case of the preceding exercise, so the solution given above suffices.

<u>10-24</u>

(a) The desired equations follow immediately from the parallel-axis theorem in the form of Eq. 10-17.

(b) The period T_1 for oscillation about knife-edge 1 is given by

$$T_1 = \frac{1}{\nu_1} = 2\pi\sqrt{\frac{G_1^2}{D_1 g}} = 2\pi\sqrt{\frac{G_o^2 + D_1^2}{D_1 g}}$$

Similarly,

$$T_2 = \frac{1}{\nu_2} = 2\pi\sqrt{\frac{G_2^2}{D_2 g}} = 2\pi\sqrt{\frac{G_o^2 + D_2^2}{D_2 g}}$$

(c) Obtaining the desired results is simply a matter of careful algebraic manipulation. Using the above equations, we find

$$T_1^2 \pm T_2^2 = \frac{4\pi^2}{g}\left[\frac{G_o^2 + D_1^2}{D_1} \pm \frac{G_o^2 + D_2^2}{D_2}\right]$$

$$= \frac{4\pi^2}{g}\left[\frac{G_o^2 D_2 + D_1^2 D_2 \pm G_o^2 D_1 \pm D_1 D_2^2}{D_1 D_2}\right]$$

$$= \frac{4\pi^2}{g}\left[\frac{\pm G_o^2(D_1 \pm D_2) + (D_1 \pm D_2)D_1 D_2}{D_1 D_2}\right]$$

$$= \pm\frac{4\pi^2}{g}\left(\frac{G_o^2}{D_1 D_2} \pm 1\right)(D_1 \pm D_2)$$

In the above equations, either all of the upper signs must be used or else all of the lower signs. This gives the two equations desired:

$$\frac{T_1^2 + T_2^2}{D_1 + D_2} = \frac{4\pi^2}{g}\left(\frac{G_o^2}{D_1 D_2} + 1\right)$$

and

$$\frac{T_1^2 - T_2^2}{D_1 - D_2} = \frac{-4\pi^2}{g}\left(\frac{G_o^2}{D_1 D_2} - 1\right)$$

(d) In the first printing of the text, there was a typographical error in the equation given in the exercise. The second term on the right side of the equation should be added to the first term, not subtracted from it. By adding the last two displayed equations of part (c), we obtain

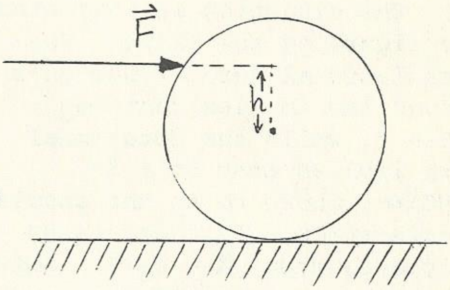
$$\frac{T_1^2 + T_2^2}{D_1 + D_2} + \frac{T_1^2 - T_2^2}{D_1 - D_2} = \frac{8\pi^2}{g}$$

(e) If the pendulum is asymmetrical (that is if the bob is sufficiently massive compared to the rod), then it is possible to move the bob until $T_1 \simeq T_2$, even though $D_1 \neq D_2$. (In actual practice, D_1 and D_2 differ by a factor of about 4.) Under these conditions, the second term in the equation for $8\pi^2/g$ is much smaller in magnitude than the first term, and can be ignored. Then

$$g \simeq \frac{1}{8\pi} \frac{(D_1 + D_2)}{(T_1^2 + T_2^2)} = \frac{D_1 + D_2}{16\pi T^2}$$

where T is the common value of T_1 and T_2, and where $D_1 + D_2$ is the accurately measurable distance between the knife edges. Under these conditions, we can expect to achieve the maximum accuracy obtainable with the device.

10-25



The situation is shown in the figure at right. When the ball is struck, a horizontal force of arbitrarily large magnitude F (and arbitrarily short duration δt) is applied. Since any static frictional force supplied by the table is limited to μMg, the only sure way to avoid slippage is to create a situation in which zero frictional force is needed. Let v_o be the speed and ω_o be the angular speed just after the impulse $F\delta t$ has been delivered. Then we must have

$$F\delta t = Mv_o$$

and

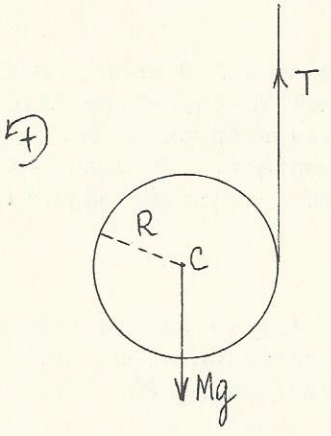
$$h(F\delta t) = I_c\omega_o$$

where $\omega_o = v_o/R$. Since billiard balls are homogeneous, the moment of inertia about the center of the ball is given by $I_c = 2MR^2/5$. Combining these equations, we find

$$h(Mv_o) = (2MR^2/5)(v_o/R)$$

which leads to $h = 2R/5$.

10-26



(a) The situation is shown at right. Since the center of mass of the spool is not accelerating, the tension T in the string must be equal to the weight: $T = Mg$.

(b) Taking torques about the central axis of the spool, we have

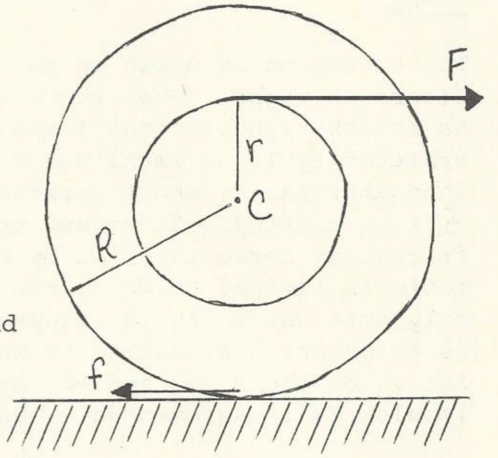
$$I_C\alpha = (-Mg)(0) + TR = \underline{MgR}$$

where $I_C = \frac{1}{2}MR^2$ is the moment of inertia of the homogeneous spool. Therefore the angular acceleration is given by $\alpha = \underline{2g/R}$.

(c) The length of string unwound when the cylinder turns through angle $\theta(t)$ equals $R\theta(t)$. Therefore the upward linear acceleration a of any part of the string must be given by $a = R\alpha = \underline{2g}$.

<u>10-27</u>

(a) The situation is indicated in the figure at the right. The translational form of Newton's second law implies that $Ma_C = F - f$, while the rotational form implies that $Fr + fR = MG_C^2\alpha$, where α is the angular acceleration. If there is no slippage, then $\alpha = a_C/R$. Solving these three equations for f, we find



$$f = \frac{F(G_C^2 - rR)}{(G_C^2 + R^2)}$$

as desired.

(b) When $r \equiv r_o = G_C^2/R$, the frictional force vanishes.

(c) When $r > r_o$, we find that $f < 0$; that is, the frictional force is directed toward the right, with magnitude equal to $F[(rR - G_C^2)/(G_C^2 + R)]$.

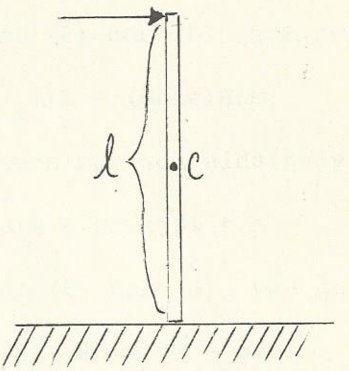
(d) The translational acceleration is given by

$$a_C = \frac{F - f}{M} = \frac{F}{M}\left[1 + \frac{(rR - G_C^2)}{(G_C^2 + R^2)}\right]$$

which exceeds F/M when $r > r_o = G_C^2/R$. This occurs because the frictional force can be directed toward the right. (A frictional force always opposes the <u>relative</u> motion of the two surfaces that are in contact. It does not necessarily oppose the translational motion of a spinning object.)

<u>10-28</u>

(a) The figure at the top of the next page displays the situation and the notation. Let the impulse $\hat{x}\int F_x dt$ delivered by the blow be represented by $P\hat{x}$. From the translational and rotational forms of Newton's second law, we find that $v_c = P/M$ and $I_c \omega_c = P\ell/2$. Here v_c is the rightward velocity of the center of mass after the blow and ω_c is the (positive clockwise) angular velocity of the rod after the blow. Using the fact that $I_c = M\ell^2/12$, we find that $\omega_c \ell = 6P/M$. Therefore $v_c = \omega_c \ell/6$. Since $\omega_c = 3.00$ rad/s and $\ell = 1.00$ m, we find v_c = 0.500 m/s.



(b) Just after the blow is struck, a point on the rod a distance d from the struck end has a rightward velocity v $= v_c + \omega_c[(\ell/2) - d)]$. We find that v = 0 for d $= (\ell/2) + (v_c/\omega_c)$. With $v_c = \omega_c \ell/6$, we find $d = (\ell/2) + (\ell/6) = 2\ell/3$. Since $\ell = 1$ m, we find that the point located 2/3 m from the struck end is initially stationary. This point is called the "center of percussion" of the rod with respect to the top end. (The location of the center of percussion depends upon the site of the blow.)

10-29

The cylinder is assumed homogeneous. Referring to Fig. 10E-29, and letting θ represent the incline angle, we find that the translational equilibrium of the cylinder requires that

$$F + \mu_k Mg \cos\theta - Mg \sin\theta = 0 \qquad (1)$$

where F is the string tension. Notice that the frictional force on the cylinder is directed up the incline, since it must oppose the relative motion between the incline and the cylinder's surface. The angular acceleration α of the cylinder is governed by the equation

$$FR - (\mu_k Mg \cos\theta)R = I_c\alpha = \frac{1}{2} MR^2\alpha \qquad (2)$$

where we have taken a positive-clockwise convention. The translational acceleration a of the hanging object is governed by

$$mg - F = ma \qquad (3)$$

The cylinder slips on the incline, but it cannot slip on the string, so a and α are related by

$$a = R\alpha \qquad (4)$$

Solving Eq. (1) for the string tension F, we obtain

$$F = Mg(\sin\theta - \mu_k \cos\theta) \qquad (5)$$

Using Eqs. (4) and (5) in Eq. (2), we find that

$$MgR(\sin\theta) - 2\mu_k \cos\theta) = \frac{1}{2} MRa$$

Solving this for the acceleration, we have

$$a = 2g(\sin\theta - 2\mu_k \cos\theta) \qquad (6)$$

Using Eqs. (6) and (5) in Eq. (3), we obtain

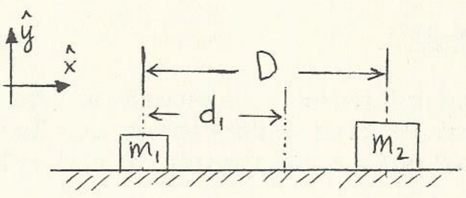
$$mg - Mg(\sin\theta - \mu_k \cos\theta) = 2mg(\sin\theta - 2\mu_k \cos\theta)$$

Solving this equation for the mass ratio (M/m), we find

$$\frac{M}{m} = \frac{1 - 2(\sin\theta - 2\mu_k \cos\theta)}{(\sin\theta - \mu_k \cos\theta)} \qquad (7)$$

Inserting the numerical values $\theta = 30^\circ$ and $\mu_k = 0.25$ into Eqs. (6) and (7), we obtain $a = \underline{0.134g}$ and $(M/m) = \underline{3.05}$.

10-30



(a) The initial configuration is as shown at right. The center of mass of the system is located between body 1 and body 2, at a distance d_1 from body 1, where $m_1d_1 = m_2(D - d_1)$. With D = 1 m, m_1 = 1 kg, and m_2 = 2 kg, we find that d_1 = 2/3 m.

(b) Applying Newton's second law, we obtain $a_2 = F_2/m_2$ = (2 N) ÷ (2 kg) = $\underline{1.00 \text{ m/s}^2 \text{ rightward}}$.

(c) The constant-acceleration kinematic equations give $s_2 = \frac{1}{2}a_2t^2 = 0.5(1.00)(1.00)^2 = \underline{0.50 \text{ m}}$.

(d) As before, we write $m_1d_1' = m_2(D' - d_1')$, so that $d_1' = 2D'/3 = 2(1.5 \text{ m})/3 = \underline{1.00 \text{ m}}$.

(e) The rightward displacement is 1 - (2/3) = 1/3 m.

(f) The x coordinate x_c of the center of mass is given by $x_c = (m_1x_1 + m_2x_2)/(m_1 + m_2) = (x_1 + 2x_2)/3$. Therefore $a_c = (a_1 + 2a_2)/3$. Since $a_1 = 0$, $a_c = 2a_2/3 = \underline{2/3 \text{ m/s}^2 \text{ rightward}}$.

(g) In this case $a = F/(m_1 + m_2) = 2 \text{ N}/(3 \text{ kg}) = \underline{2/3 \text{ m/s}^2 \text{ rightward}}$.

(h) The center of mass of any system moves with an acceleration $\vec{a}_c = \vec{F}/M$, where $\vec{F} \equiv \sum \vec{F}_{ext}$ is the resultant of all external forces acting on the system, and $M \equiv \sum_n m_n$ is the total mass of the system.

10-31

As the wheel is brought from rest to operating speed, it undergoes a constant angular acceleration $\alpha = \omega_f/(\Delta t) = (400\,\pi/60)/(3.0) = 6.981\ \text{rad/s}^2$. The moment of inertia of the wheel is given by $I = \frac{1}{2}MR^2 = (0.50)(100)(0.32)^2 = 5.120\ \text{kg m}^2$. Therefore the (constant) torque exerted is $T = I\alpha = 35.74\ \text{N}\cdot\text{m}$. Since the torque is constant, the integral simplifies:

$$W = \int_{\varphi_i}^{\varphi_f} \vec{T}\cdot d\vec{\varphi} = T\int_{\varphi_i}^{\varphi_f} d\varphi = T\cdot(\varphi_f - \varphi_i)$$

The angular displacement is given by $\varphi_f - \varphi_i = \frac{1}{2}\alpha(\Delta t)^2 = \frac{1}{2}\omega_f \Delta t = 31.41$ rad. Therefore $W = (35.74)(31.41) = 1.12 \times 10^3$ J $=$ $\underline{1.1 \times 10^3\ \text{J}}$, in agreement with the results obtained in Example 10-8.

10-32

We let I_o be the moment of inertia of the lecturer plus the stool, but excluding the two dumbbells. We assume that I_o does not change during the experiment. Since no torques can be exerted about the vertical axis, the vertical component of the total momentum of the system is conserved. Therefore $(I_o + 2mR_1^2)\,\omega_1 = (I_o + 2mR_2^2)\,\omega_2$. Solving for ω_2, we obtain

$$\omega_2 = \left[\frac{(I_o + 2mR_1^2)}{(I_o + 2mR_2^2)}\right]\omega_1$$

The initial and final kinetic energies are $K_1 = \frac{1}{2}(I_o + 2mR_1^2)\,\omega_1^2$, and

$$K_2 = \frac{1}{2}(I_o + 2mR_2^2)\,\omega_2^2$$

$$= \frac{1}{2}\frac{(I_o + 2mR_1^2)^2}{(I_o + 2mR_2^2)}\,\omega_1^2$$

$$= \left[\frac{(I_o + 2mR_1^2)}{(I_o + 2mR_2^2)}\right]\cdot K_1 > K_1$$

The additional kinetic energy $(K_2 - K_1)$ appears in the system by virtue of the work done by the lecturer as he/she pulls in the weights.

10-33

(a) The moment of inertia I_o of the top is given by $I_o = \frac{1}{2} m_o r_o^2$. The work required to set the top spinning with angular speed ω_s is equal to the spin kinetic energy $\frac{1}{2} I_o \omega_s^2$. For the given numerical values, we find $I_o = (0.50)(0.050)(2.0 \times 10^{-2})^2 = 10^{-5}$ kg·m². The work required is $(0.5)(10^{-5})(200\pi)^2 = \underline{1.97 \text{ J}}$.

(b) According to Eq. (10-36), the angular speed of steady precession is

$$\omega_p = \frac{dg}{G_o^2} \frac{1}{\omega_s}$$

where $G_o^2 = I_o/m_o = \frac{1}{2} r_o^2$. With the numerical values given, we find

$$\omega_p = \frac{(3.00 \times 10^{-2})(9.80)}{(0.5)(2.00 \times 10^{-2})^2} \frac{1}{200\pi}$$

$$= \underline{2.34 \text{ rad/s}}$$

The ratio of this angular speed to ω_s is $(\omega_p/\omega_s) = 3.72 \times 10^{-3}$. The assumption that $(\omega_p/\omega_s) \ll 1$ is fulfilled. [NOTE: As can be seen in the solution to Exercise 10-46, the actual criterion for the validity of Eq. (10-36) is $(4d^2/G^2)(\omega_p/\omega_s) \ll 1$, which is stricter than $(\omega_p/\omega_s) \ll 1$. The more complicated formula given in Exercise 10-46 is valid for all values of the ratio (ω_p/ω_s). In the present case, $(4d^2/G^2) = 18$, so that $(4d^2/G^2) \times (\omega_p/\omega_s) = 6.7 \times 10^{-2}$. Thus the strict criterion is satisfied; in fact, use of the exact formula indicates that Eq. (10-36) provides a value for ω_p that is accurate to within 2%.]

10-34

Referring to Fig. 10E-34, if we take the lowest point on the disk as the reference level for gravitational potential energy, the initial total energy is given by

$$E_i = K_i + U_i = U_i$$

$$= MgR + Mg(2R) = 3MgR$$

When the object passes beneath the pivot, the potential energy is given by

$$U_f = MgR + Mg \cdot 0 = MgR$$

The final kinetic energy of the system is given by

$$K_f = \frac{1}{2} I_d \omega_f^2 + \frac{1}{2} M(\omega_f R)^2 = \frac{3}{4} M \omega_f^2 R^2$$

since $I_d = \frac{1}{2}MR^2$ is the moment of inertia of the uniform disk. The system is conservative, so that

$$K_f = \frac{3}{4} M \omega_f^2 R^2 = E_i - U_f = 3MgR - MgR$$

Solving for ω_f, we obtain $\underline{\omega_f = \sqrt{8g/3R}.}$

10-35

(a) Since the sphere rolls without slipping, its spin kinetic energy is proportional to its translational kinetic energy. Furthermore, the loop has constant curvature. Therefore the minimum elevation h is determined by the condition that the sphere remain in contact with the track at the top of the loop (as shown in Fig. 10E-35). We use the subscripts i and f to denote the values at release and at the top of the loop, respectively. When the elevation has its minimum value h, the centripetal acceleration at the top of the loop is supplied by the sphere's weight:

$$\frac{mv_f^2}{(R - r)} = mg \qquad (1)$$

Notice that the center of the sphere follows a vertical circular path of radius R - r. The kinetic energy at the top of the loop is given by $K_f = \frac{1}{2}I_c \omega_f^2 + \frac{1}{2}mv_f^2 = \frac{1}{5}mr^2\omega_f^2 + \frac{1}{2}mv_f^2$. Since the sphere rolls without slipping, we must have $\omega_f = v_f/r$, so that

$$K_f = \frac{7mv_f^2}{10} = \frac{7}{10} mg(R - r) \qquad (2)$$

Here we have used Eq. (1). If we take the top of the loop to be the reference level for gravitational potential energy, we find

$$U_i = mg(r + h) \qquad (3)$$

and

$$U_f = -mgr \qquad (4)$$

Since the sphere is released from rest, the initial kinetic energy $K_i = 0$. Using Eqs. (2) - (4), the conservation of total energy implies that

$$mg(r + h) = \frac{7mg}{10} (R - r) - mgr$$

or

$$\underline{h = \frac{1}{10} (7R - 27r)}$$

(b) If $r > 7R/27$, the above equation assigns a negative value to

h. That this is possible can be seen by observing that when the sphere is at the top of the loop, its center is a distance $2r + h$ below its initial height. Since we must have $r \leq R$, it is easy to show that

$$2r + h = \frac{7}{10}(R - r)$$

is a positive quantity (or zero). So even though h can be negative, the sphere is lower at the top of the loop than it is at release.

10-36

(a) Just before the string becomes taut, the cylinder is still at rest: nothing has caused it to begin turning. That is, $\omega_0 = 0$. Since the string has exerted no force on the body of mass m, it has fallen freely: $v_o = \sqrt{2gh}$. The kinetic energy of the system if given by $K_o = \frac{1}{2}mv_o^2 + \frac{1}{2}I_c\omega_o^2 = mgh$.

(b) The string is assumed to be inextensible, so $v_1 = \omega_1 R$. Furthermore, the angular momentum of the system must be conserved, so we have $L_1 \equiv mv_1R + \frac{1}{2}MR^2\omega_1 = L_o$, where $L_o = mv_oR = m\sqrt{2gh}\,R$. Solving for the angular speed, we find

$$\omega_1 = \frac{v_o}{R[1 + (M/2m)]} = \frac{\sqrt{2gh}}{R[1 + (M/2m)]}$$

The corresponding speed is given by

$$v_1 = \omega_1 R = \frac{v_o}{[1 + (M/2m)]} = \frac{\sqrt{2gh}}{[1 + (M/2m)]}$$

The final kinetic energy K_1 is given by

$$K_1 = \frac{1}{2}mv_1^2 + \frac{1}{2}I_c\omega_1^2$$

$$= \frac{1}{2}mv_1^2 + \frac{1}{2}\cdot\left(\frac{1}{2}MR^2\right)\left(\frac{v_1^2}{R^2}\right)$$

$$= \frac{1}{2}\left(m + \frac{M}{2}\right)\cdot v_1^2 = \frac{1}{2}\frac{mv_o^2}{[1 + (M/2m)]}$$

$$= \frac{K_o}{[1 + (M/2m)]}$$

(c) We notice that $K_1 < K_o$. The "lost kinetic energy" is converted to heat energy and/or potential energy of deformation in the string and/or in the two massive objects. (The "tweaking" of the string leads to a dissipation of macroscopic kinetic energy that is analogous to the energy loss in completely inelastic two-body collisions.)

(d) If $M = m$, $K_1 = 2K_o/3$, so the fraction lost is $(K_1 - K_o)/K_o$ $= \underline{1/3}$.

10-37

(a) In neither descent is any work done against friction. If h is the height of the incline, the final kinetic energy $K_f = Mgh$, where M is the mass of the sphere. For the rolling descent, the kinetic energy is given by

$$K_f = \frac{1}{2} Mv_o^2 + \frac{1}{2} I_c\omega_o^2 = \frac{1}{2} Mv_o^2 + \frac{1}{2} MG_c^2(v_o^2/R^2)$$

$$= \frac{1}{2} Mv_o^2(1 + G_c^2/R_o^2)$$

For the sliding descent, we have

$$K_f = \frac{1}{2} M(5v_o/4)^2 = \frac{1}{2} Mv_o^2(\frac{25}{16})$$

From these equations we find $G_c^2 = 9R_o^2/16$, or $G_c = \underline{3R_o/4}$.

(b) Let the density of the solid portion be ρ_o. Then the mass of the hollow sphere is given by

$$M = \frac{4\pi}{3}(\rho_o R_o^3 - \rho_o R_i^3)$$

and the moment of inertia is given by

$$I_c = \frac{2}{5}(\frac{4\pi}{3}\rho_o R_o^3)R_o^2 - \frac{2}{5}(\frac{4\pi}{3}\rho_o R_i^3)R_i^2$$

The radius of gyration G_c satisfies the equation

$$G_c^2 = \frac{I_c}{M} = \frac{2}{5}\frac{\left(R_o^5 - R_i^5\right)}{\left(R_o^3 - R_i^3\right)}$$

$$= \frac{2}{5} R_o^2 \frac{[1 - (R_i/R_o)^5]}{[1 - (R_i/R_o)^3]}$$

In part (a) it was established that $G_c^2 = 9R_o^2/16$. Therefore we have

$$\frac{45}{32} = \frac{1 - (R_i/R_o)^5}{1 - (R_i/R_o)^3}$$

which leads to

$$\frac{13}{45} = \left(\frac{R_i}{R_o}\right)^3 \left[1 - \frac{32}{45}\left(\frac{R_i}{R_o}\right)^2\right]$$

A simple search algorithm rather quickly yields a solution good to three decimal places: R_i/R_o = 0.823. The corresponding volume ratio $V_i/V_o = (R_i/R_o)^3$ = 0.557.

10-38

The spin energy K_s stored in the flywheel is $K_s = \frac{1}{2} I \omega_s^2$. If this is used to do work against a frictional force F, the stored spin energy will be consumed after the car has traveled a distance d such that $Fd = K_s$. Therefore $d = (\frac{1}{2} I \omega_s^2)/F$. In the present exercise, we are given $I = \frac{1}{2}MR^2 = \frac{1}{2}(200)(0.50)^2 = 25\ kg{\cdot}m^2$. Also, $\omega_s = 2\pi(2 \times 10^4)/60 = 2.094 \times 10^3$ rad/s. Therefore $\frac{1}{2} I \omega_s^2 = 5.483 \times 10^7$ J. Since $F = 0.1Mg = (0.1)(1000)(9.80) = 980$ N, we find $d = (5.483 \times 10^7)/(0.980 \times 10^3) = 5.595 \times 10^4$ m = 56.0 km.

10-39

(a) The moment of inertia I_u of a uniform sphere with the same mass and radius as the earth is given by $I_u = \frac{2}{5}(5.98 \times 10^{24}) \times (6.37 \times 10^6)^2 = 9.71 \times 10^{37}\ kg{\cdot}m^2$. The actual value I_e is given as $I_e = 8.04 \times 10^{37}\ kg{\cdot}m^2$ so I_e/I_u = 0.828. The actual moment of inertia is smaller than that of a uniform sphere because the density within the earth increases with depth, which puts relatively more material at smaller radial distances.

(b) The spin angular momentum $\vec{L}_s = L_s\hat{s}$ where $\hat{s}$ is a unit vector parallel to the earth's rotation axis and directed from south toward north. The magnitude $L_s = I_e\omega_s$, where $\omega_s = 2\pi/(8.62 \times 10^4\ s) = 7.29 \times 10^{-5}$ rad/s. Therefore $L_s = 5.86 \times 10^{33}\ kg{\cdot}m^2/s$.

(c) The orbital angular momentum $\vec{L}_r = L_r\hat{r}$, where r is a unit vector at right angles to the earth's orbit. (The direction of $\hat{r}$ is given by the right-hand rule. Astronomically speaking, r is directed toward the north ecliptic pole.) The magnitude $L_r = M_e v_e r_o$, where r_o is the orbital radius, and $v_e = 2\pi r_o/T_o$ is the orbital speed. Using the values given for r_o and the orbital period T_o, we find $v_e = 2.98 \times 10^4$ m/s = 29.8 km/s. Then $L_r = (5.98 \times 10^{24})(2.98 \times 10^4)(1.50 \times 10^{11}) = 2.68 \times 10^{40}\ kg{\cdot}m^2/s$.

(d) The total angular momentum $\vec{L}_{tot} = \vec{L}_s + \vec{L}_r$. Its magnitude is given by

$$L_{tot} = \left[L_s^2 + L_r^2 + 2L_r L_s \hat{r}\cdot\hat{s}\right]^{1/2}$$

The angle θ between $\hat{r}$ and $\hat{s}$ (called the obliquity of the ecliptic), is about 23.5°, so that $\hat{r}\cdot\hat{s} \equiv \cos\theta = 0.917$. (Since no information on $\hat{r}$ and $\hat{s}$ is provided in the exercise, it would be reasonable for a student to assume that $\hat{r}$ and $\hat{s}$ are parallel.) However, the fact that $L_s/L_r = 2.19 \times 10^{-7}$ means that one would obtain

L_s/L_{tot} = $\underline{2.19 \times 10^{-7}}$, no matter what value is taken for the obliquity of the ecliptic.

(e) The spin kinetic energy $K_s = \frac{1}{2} I_e \omega_s^2 = \underline{2.14 \times 10^{29}\ J}$.

(f) The kinetic energy of orbital motion is $K_o = \frac{1}{2} M_e v_e^2 = \underline{2.66 \times 10^{33}\ J}$.

(g) The ratio $K_s/(K_s + K_o) = \underline{8.06 \times 10^{-5}}$.

<u>10-40</u>

(a) The total mass M is given by the integral of the surface density σ over the area of the shell: $M = \int \sigma\, dA$. Since the shell is homogeneous, the surface density σ is constant. Therefore $M = \sigma \int dA = \sigma A = 4\pi R^2 \sigma$. This implies that $\underline{\sigma = M/4\pi R^2}$.

(b) Referring to Fig. 10E-40, the hoop shown has a radius R sinθ and therefore a circumference of $2\pi R \sin\theta$. The width of the hoop is Rdθ, so the area $dA = (2\pi R \sin\theta)(R d\theta)$, and the mass $dM = \sigma dA = (M/4\pi R^2)(2\pi R^2 \sin\theta d\theta) = \underline{(M/2) \sin\theta d\theta}$.

(c) We use dI to denote the moment of inertia of the hoop about the axis ZZ'. The perpendicular distance from the axis to each part of the hoop is R sinθ. Therefore $dI = (R \sin\theta)^2 dM = \underline{(MR^2/2) \sin^3\theta d\theta}$.

(d) The entire shell is swept out as <u>θ ranges from 0 to π</u>.

(e) The moment of inertia of the entire shell is the sum of the moments of inertia of the various hoops:

$$I = \int dI_c = \int_0^{\pi} (MR^2/2) \sin^3\theta d\theta$$

$$= (MR^2/2) \int_0^{\pi} (1 - \cos^2\theta) \sin\theta d\theta$$

$$= (MR^2/2) \left[-\cos\theta \Big|_0^{\pi} + \int_0^{\pi} \cos^2\theta (-\sin\theta d\theta) \right]$$

$$= \frac{MR^2}{2} \left\{ \left[-(-1) - (-1) \right] + \frac{\cos^3\theta}{3} \Big|_0^{\pi} \right\}$$

$$= \frac{MR^2}{2} \left[2 + \frac{(-1)^3}{3} - \frac{(1)^3}{3} \right]$$

$$= \frac{MR^2}{2} \left(\frac{4}{3} \right) = \underline{\frac{2MR^2}{3}}$$

This corresponds to a gyration radius G such that $G^2 = 2R^2/3$, in agreement with Table 10-1.

10-41

(a) The mass of a sphere is the integral $\int \rho\, dV$ where ρ is the mass density. For a homogeneous sphere ρ is a constant, so $M \equiv \int \rho\, dV = \rho \int dV = \rho V = (4\pi R^3/3)\rho$. Therefore $\rho = M/(4\pi R^3/3)$, as desired.

(b) The surface area of a spherical shell of radius r is $4\pi r^2$. If the shell has thickness $dr \ll r$, the volume of the shell is simply the surface area times the thickness: $dV = 4\pi r^2 dr$. Alternatively, the volume of a thick shell (which extends from r to $r + \Delta r$) is given by

$$V = (4\pi/3)(r + \Delta r)^3 - (4\pi/3)r^3$$

$$= 4\pi/3[r^3 + 3r^2\Delta r + 3r(\Delta r)^2 + (\Delta r)^3 - r^3]$$

$$= 4\pi r^2 \Delta r[1 + (\Delta r/r) + \tfrac{1}{3}(\Delta r/r)^2]$$

which is given by $4\pi r^2 \Delta r$ to any desired degree of accuracy for sufficiently small values of $(\Delta r/r)$. Since the volume of the shell is $dV = 4\pi r^2 dr$, the mass of the shell is $dM = \rho(4\pi r^2 dr) = (3M/R^3)r^2 dr$.

(c) As listed in Table 10-1 and confirmed in Exercise 10-40, the moment of inertia of a spherical shell is given by $dI = (2/3)(dM)r^2 = (2M/R^3)r^4 dr$.

(d) The entire sphere is swept out when r ranges from 0 to R.

(e) The moment of inertia of the sphere is the sum of the moments of inertia of the various shells:

$$I = \int dI = \frac{2M}{R^3}\int_0^R r^4 dr$$

$$= \frac{2M}{R^3}\cdot\frac{R^5}{5} = \frac{2MR^2}{5}$$

This corresponds to a gyration radius G such that $G^2 = 2R^2/5$, in agreement with Table 10-1.

10-42

(a) Because the particles are of equal mass, the center of mass is located midway between them -- that is, at O. Since point O is on the axis AB, it is motionless.

(b) Referring to Fig. 10E-42, we see that the "upper" particle is moving directly up out of the plane of the diagram with speed

$v_u = \omega[(d/2)\sin\gamma]$. Similarly, the "lower" particle is moving directly down into the plane of the diagram with speed $v_\ell = \omega[(d/2)\sin\gamma] = v_u$. As measured from O, the particles have instantaneous radius vectors of equal length d/2 but opposite in direction. Since $\vec{r}_u = -\vec{r}_\ell$ and $\vec{v}_u = -\vec{v}_\ell$, the angular momenta of the two particles must be equal. Applying the right hand rule, we conclude that the instantaneous total angular momentum lies in the plane of the diagram and is directed as shown. Since the radius vectors are perpendicular to the velocity vectors, $|\vec{L}| = 2m|\vec{r}|\cdot|\vec{v}| = 2m(d/2)\omega(d/2)\sin\gamma = \frac{1}{2}m\omega d^2\sin\gamma$.

(c) Equation 10-28 states that the total angular momentum of a system about any chosen origin can be written as a sum of two vectors: (1) the angular momentum about an origin located at the center of mass, and (2) the angular momentum due to motion of the center of mass. In the text following Eq. (10-28), it is shown that the latter term is given by $\vec{r} \times M\vec{v}$, where $\vec{r}$ is the instantaneous position of the center of mass, $\vec{v} \equiv dr/dt$, and M is the total mass of the system. In the present exercise, $\vec{v} = \vec{0}$, so the second term in Equation 10-28 vanishes for all choices of the coordinate origin (as long as it is not translating with respect to the shaft AB.)

(d) The vector $\vec{L}_\parallel = L_\parallel\hat{\omega}$, where $L_\parallel = L\cos(\pi/2 - \gamma) = L\sin\gamma = \frac{1}{2}m\omega d^2\sin^2\gamma$. The vector $\vec{L}_\perp$ has magnitude $L_\perp = \sqrt{L^2 - L_\parallel^2} = \frac{1}{2}m\omega d^2\sin\gamma\cos\gamma$.

(e) The component of $\vec{L}$ parallel to the axis does not change with time because that component depends on the magnitudes and relative orientations of $\vec{r}_\perp$ and $\vec{v} = \vec{v}_\perp = \vec{\omega} \times \vec{r}_\perp$. Although both $\vec{r}_\perp$ and $\vec{v}_\perp$ rotate, they do not vary in magnitude or in <u>relative</u> orientation. By contrast, careful examination of Fig. 10E-42 shows that $\vec{L}$ lies in the instantaneous plane containing the rod and the axis AB. For the instant shown, $\vec{L}_\perp$ lies in the plane of the diagram and is directed toward the bottom of the page. Its rate of change $d\vec{L}_\perp/dt$ is directed into the page at the instant depicted. The magnitude of $\vec{L}_\perp$ does not change, so $d\vec{L}_\perp/dt$ must be perpendicular to $\vec{L}_\perp$ at every instant. Indeed, since $\vec{L}$ sweeps around every time the shaft rotates once, we have $d\vec{L}/dt = \vec{\omega} \times \vec{L}$. Since $d\vec{L}_\parallel/dt = \vec{0}$ and $\vec{\omega} \times \vec{L}_\parallel = \vec{0}$, this reduces to $d\vec{L}_\perp/dt = \vec{\omega} \times \vec{L}_\perp$. The magnitude of $d\vec{L}_\perp/dt$ is $\underline{\frac{1}{2}m\omega^2 d^2\sin\gamma\cos\gamma}$.

(f) We adopt a Cartesian system with $\hat{x}$ directed from A toward B, with $\hat{y}$ in the plane of the diagram (and directed toward the top of the page), and with $\hat{z}$ directed straight up out of the plane of the diagram. Then $\vec{\omega} = \omega\hat{x}$, and $\vec{L} = L_\parallel\hat{x} - L_\perp\hat{y}$ for the instant shown. The vector $d\vec{L}_\perp/dt$ is given by $\vec{\omega} \times \vec{L} = -\omega L_\perp\hat{z}$ for the instant shown. This change implies the existence of a net torque provided by the bearings at A and B. Denoting the forces exerted by the bearings as $\vec{F}_A$ and $\vec{F}_B$, the torque about point O is

$\vec{T} = \vec{r}_A \times \vec{F}_A + \vec{r}_B \times \vec{F}_B$. Since the center of mass of the system is not accelerating, we must have $\vec{F}_A = -\vec{F}_B$. Therefore the torque is given by $-\omega L_\perp \hat{z} = \vec{T} = (\vec{r}_B - \vec{r}_A) \times \vec{F}_B$. Since $\vec{r}_B - \vec{r}_A = h\hat{x}$, we find that $\vec{F}_B \cdot \hat{z} = 0$ and $\vec{F}_B \cdot \hat{y} = -(m\omega^2 d^2/2h) \sin\gamma \cos\gamma$. It is not possible to determine $\vec{F}_B \cdot \hat{x}$, but we do know that the vector $\vec{F}_A = -\vec{F}_B$. If we suppose that $\vec{F}_B \cdot \hat{x} = -\vec{F}_A \cdot \hat{x} = 0$, then for the instant depicted, we have

$$\vec{F}_A = +\hat{y}(m\omega^2 d^2/2h) \sin\gamma \cos\gamma$$

and

$$\vec{F}_B = -\hat{y}(m\omega^2 d^2/2h) \sin\gamma \cos\gamma$$

These results are independent of the position of O along the shaft AB. Because the forces $\vec{F}_B$ and $\vec{F}_A$ are equal and opposite, their torques about O lie in the same direction, and the entire shaft length h is effective as a lever arm, regardless of the position of O. (The forces $\vec{F}_A$ and $\vec{F}_B = -\vec{F}_A$ constitute a couple.)

<u>10-43</u>

It is possible to determine the total mass M and moment of inertia I without removing any material from the object. To decide whether this will allow us to determine the size and density of the two parts, we denote the radius of the boundary by r and the inner and outer densities by ρ_i and ρ_o. Then the mass is given by

$$M = \frac{4\pi}{3}\rho_i r^3 + \frac{4\pi}{3}\rho_o (R^3 - r^3)$$

$$= \frac{4\pi}{3}(\rho_i - \rho_o) r^3 + \frac{4\pi}{3}\rho_o R^3$$

The moment of inertia is given by

$$I = \frac{8\pi}{15}(\rho_i - \rho_o) r^5 + \frac{8\pi}{15}\rho_o R^5$$

These two equations can be solved for (r/R) in terms of M, I, R, and ρ_o:

$$\frac{r}{R} = \left\{ \frac{\left[\left(\frac{15I}{8\pi}\right) - \rho_o R^5\right]}{\left[\left(\frac{3M}{4\pi}\right) - \rho_o R^3\right]} \right\}^{1/2}$$

Once r/R is determined, then ρ_i can be obtained, using the equation

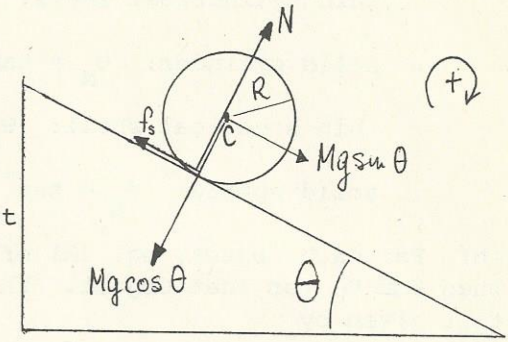
$$\rho_i = \frac{3M}{4\pi r^3} - \rho_o \left(\frac{R^3}{r^3} - 1\right)$$

Based on the last two displayed equations, if the outer density ρ_o

could be regarded as known, we could obtain values for r and ρ_i. However, unless ρ_o is known, the problem is underdetermined: we have two equations in the three unknowns r, ρ_i, and ρ_o. In this case, the sizes and densities of the two parts cannot be obtained.

10-44

(a) The general situation and the notation are shown at right. The object has mass M, radius R, and moment of inertia $I_c = MG_c^2 \equiv \beta MR^2$ about an axis through its center of mass. If the object rolls down the incline, the translational and rotational forms of Newton's second law imply that



$$Ma_c = Mg \sin\theta - f_s \qquad (1)$$

and

$$I_c \alpha_c = f_s R \qquad (2)$$

where a_c is the acceleration down along the incline, and where α_c is the angular acceleration (positive clockwise). Using $\alpha_c = a_c/R$, and solving equations (1) and (2) for a_c, we obtain

$$a_c = \frac{g \sin\theta}{(1 + \beta)} \qquad (3)$$

The required static frictional force f_s is given by

$$f_s = M(g \sin\theta - a_c) = \frac{Mg\,\beta \sin\theta}{(1 + \beta)}$$

However the maximum possible static frictional force f is

$$\max(f_s) = \mu_s N = \mu_s Mg \cos\theta$$

Therefore, in order for the object to roll down the incline, the angle θ must satisfy

$$\frac{Mg\,\beta \sin\theta}{(1 + \beta)} \leq \mu_s Mg \cos\theta$$

This implies that $\theta \leq \theta_M$, where

$$\theta_M = \tan^{-1}\left[\frac{\mu_s(1 + \beta)}{\beta}\right]$$

Referring to Table 10-1, we find the values of β for the various objects. For a thin cylindrical shell, $\beta = 1$. For a solid cylinder, $\beta = 1/2$. For a thin spherical shell, $\beta = 2/3$. For a solid sphere, $\beta = 2/5$. Therefore, the maximum angles consistent with rolling are as follows.

thin cylindrical shell: $\theta_M = \tan^{-1}(2\mu_s)$

solid cylinder: $\theta_M = \tan^{-1}(3\mu_s)$

thin spherical shell: $\theta_M = \tan^{-1}(5\mu_s/2)$

solid sphere: $\theta_M = \tan^{-1}(7\mu_s/2)$

(b) For each object, eq. (3) of part (a) gives the acceleration when $\theta \leq \theta_M$ for that object. Then the distance traveled in time t is given by

$$s_c(t) = \frac{1}{2} a_c t^2 = \frac{1}{2}\left[\frac{g \sin\theta}{(1+\beta)}\right] t^2 \qquad (4)$$

For the various objects, the accelerations are as follows.

thin cylindrical shell: $a_c = (g \sin\theta)/2$

solid cylinder: $a_c = (2g \sin\theta)/3$

thin spherical shell: $a_c = (3g \sin\theta)/5$

solid sphere: $a_c = (5g \sin\theta)/7$

(c) For any one of the objects, when $\theta > \theta_M$, there is slippage. The acceleration must be determined from

$$Ma_c = Mg \sin\theta - \mu_k Mg \cos\theta$$

The acceleration is then

$$a_c = g(\sin\theta - \mu_k \cos\theta) \qquad (5)$$

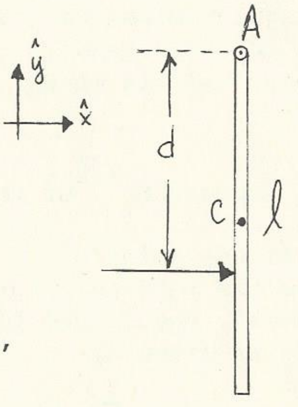
The angle θ_M above which slippage occurs, and the amount of slippage that occurs, both depend upon β. However, any of the objects that are slipping rather than rolling will have a common acceleration given by equation (5). Therefore, when θ θ_M,

$$s_c(t) = \frac{1}{2} g(\sin\theta - \mu_k \cos\theta)t^2 \qquad (6)$$

(d) With $\mu_s = 0.30$, we find the following values of θ_M. For the thin cylindrical shell, $\theta_M = \tan^{-1}(2\mu_s) = \tan^{-1}(0.60) = 31.0^\circ$. For the solid cylinder, $\theta_M = \tan^{-1}(3\mu_s) = \tan^{-1}(0.90) = 42.0^\circ$. For the thin spherical shell, $\theta_M = \tan^{-1}(5\mu_s/2) = \tan^{-1}(0.75)$

$= 36.9^\circ$. For the solid sphere, $\theta_M = \tan^{-1}(7\mu_s/2) = \tan^{-1}(1.05) = 46.4^\circ$. Since $\theta = 40^\circ$, we find that the cylindrical shell and spherical shell both slip as they descend the incline, and that they descend with a common acceleration given by equation (5): $a_C = g[\sin 40^\circ - (0.10)\cos 40^\circ] = 0.566g$. The solid cylinder rolls with the acceleration found in part (b): $a_C = [2g\sin(40^\circ)/3] = 0.429g$. The solid sphere rolls with acceleration $a_C = [5g\sin(40^\circ)/7] = 0.459g$. Therefore the objects arrive at the incline in the following order: first the two thin shells arrive simultaneously, then the solid sphere, and finally the solid cylinder.

10-45



(a) The bar is shown at the figure on the right. If we denote the impulse by $\vec{P} = \hat{x}\int F_x dt$, then if the blow is placed so that there is no impulsive force on the axle, the center of mass C of the bar begins moving to the right with speed $v_C = P/m$. Using A as the axis for computing torques, the rotational form of Newton's second law gives $Pd = I_A\omega$, where $I_A = m\ell^2/3$ is the moment of inertia of the bar about the pivot. Since the distance from A to C is $\ell/2$, we also have $v_C = \omega\ell/2$. By combining the equations, we find $m(\omega\ell/2)d = (m\ell^2/3)\omega$, which implies that $d = \underline{2\ell/3}$.

(b) The oscillation frequency is given by Eq. (10-23):

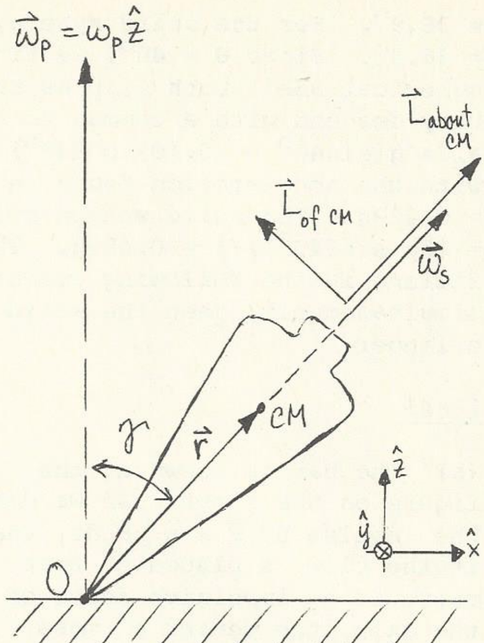
$$\nu_A = \frac{1}{2\pi}\sqrt{\frac{(\ell/2)g}{G_A^2}}$$

where $G_A^2 = I_A/m = \ell^2/3$. Therefore we obtain

$$T_A = 2\pi\sqrt{\frac{2\ell}{3g}}$$

(c) The period of a simple pendulum of length L is given by $2\pi\sqrt{L/g}$. Therefore the length of a simple pendulum with the oscillation period T_A is $L = \underline{2\ell/3}$, which is equal to the distance found in part (a).

10-46



The system is shown at the right for an instant when the top's spin axis lies in the xz plane. (The y axis is directed down into the plane of the figure.) As in Fig. 10-18, γ denotes the angle between the spin axis and the upward vertical. However, in the present case it is not possible to make the approximation that the total angular momentum $\vec{L}$ is parallel to $\vec{\omega}_s$. Instead we must base our analysis on Eq. (10-28):

$$\vec{L} = \vec{L}_{about\ CM} + \vec{L}_{of\ CM}$$

Using the (fixed) point of support O as the reference for angular momenta, the second term on the right is given by

$$\vec{L}_{of\ CM} = M\vec{r} \times \vec{v} = M\vec{r} \times (\vec{\omega}_p \times \vec{r})$$

$$= Mr^2\vec{\omega}_p - M(\vec{\omega}_p \cdot \vec{r})\vec{r} \qquad (1)$$

Here $\vec{\omega}_p = \hat{z}\,\omega_p$ is the constant precessional angular velocity; $\vec{r}$ is the vector from O to the center of mass CM, as indicated in the figure. The angular momentum about the center of mass (the "spin angular momentum") is given by

$$\vec{L}_{about\ CM} = I_s\vec{\omega}_s \qquad (2)$$

The correct kinematical equation for steady precession is

$$\frac{d\vec{L}}{dt} = \vec{\omega}_p \times \vec{L} \qquad (3)$$

while the dynamical equation is

$$\frac{d\vec{L}}{dt} = \vec{T} = \vec{r} \times (M\vec{g}) \qquad (4)$$

where $\vec{g} = -g\hat{z}$. For the instant shown in the figure, $\vec{\omega}_s = \omega_s(\hat{x}\sin\gamma + \hat{z}\cos\gamma)$ so that Eq. (2) implies that

$$\vec{L}_{about\ CM} = \hat{x}(I_s\omega_s \sin\gamma) + \hat{z}(I_s\omega_s \cos\gamma)$$

Using Eq. (1) with $\vec{r} = r(\hat{x} \sin\gamma + \hat{z} \cos\gamma)$, we obtain

$$\vec{L}_{\text{of CM}} = Mr^2[\hat{z}\omega_p - \omega_p \cos\gamma(\hat{x} \sin\gamma + \hat{z} \cos\gamma)]$$

$$= \hat{x}(-Mr^2 \omega_p \sin\gamma \cos\gamma) + \hat{z}(Mr^2\omega_p - Mr^2\omega_p \cos^2\gamma)$$

The total angular momentum $\vec{L}$ can be written as a sum of two components: one parallel to $\vec{\omega}_p$ and one perpendicular to $\vec{\omega}_p$: $\vec{L} = \vec{L}_{\parallel} + \vec{L}_{\perp} = L_{\parallel} \hat{z} + \vec{L}_{\perp}$. The parallel component is constant: $L_{\parallel} = I_s \omega_s \cos\gamma + Mr^2 \omega_p \sin^2\gamma$. For the instant shown in the figure,

$$\vec{L}_{\perp} = \hat{x}(I_s \omega_s \sin\gamma - Mr^2 \omega_p \sin\gamma \cos\gamma) \quad (5)$$

and Eq. (4) implies that the instantaneous torque is given by

$$\frac{d\vec{L}}{dt} = -\hat{y}Mgr \sin\gamma \quad (6)$$

When $\vec{L}$ is written as $\vec{L}_{\parallel} + \vec{L}_{\perp}$, Eq. (3) reduces to

$$\frac{d\vec{L}_{\perp}}{dt} = \vec{\omega}_p \times \vec{L}_{\perp} \quad (7)$$

With proper attention to the right hand rule, Eqs. (5)-(7) yield the desired equation for steady precession:

$$Mgr \sin\gamma = \omega_p(I_s\omega_s \sin\gamma - Mr^2\omega_p \cos\gamma) \quad (8)$$

Neglecting the trivial extreme case $\sin\gamma = 0$, Eq. (8) leads to the following quadratic equation for ω_p:

$$(Mr^2 \cos\gamma)\omega_p^2 - (I_s\omega_s)\omega_p + Mgr = 0 \quad (9)$$

Solving this equation for ω_p, we find the two solutions

$$\omega_{p\pm} = \frac{I_s\omega_s \pm \sqrt{(I_s\omega_s)^2 - 4M^2r^3g \cos\gamma}}{2Mr^2 \cos\gamma} \quad (10)$$

The upper sign in Eq. (10) corresponds to "fast precession", while the lower sign corresponds to the "slow precession" of interest in the present exercise. [It should also be noticed that $\omega_{p\pm}$ is real-valued only if the top is spinning sufficiently rapidly for its orientation: $\omega_s^2 \geq (4M^2r^3g \cos\gamma/I_s^2)$. The motion of the top when this condition fails is not considered here.] We simplify Eq. (10) by introducing the gyration radius G: $G^2 \equiv I_s^2/M$, and by defining

$$\omega_{po} \equiv \frac{gr}{G^2 \omega_s} \tag{11}$$

which is the asymptotically correct precession rate when $\omega_s \gg \omega_p$. Then we find that the angular velocity of the slow precession is given by

$$\omega_{p-} = \frac{g}{2\omega_{po} r \cos\gamma} \left[1 - \sqrt{1 - \left(\frac{4\omega_{po}^2 r \cos\gamma}{g} \right)} \right] \tag{12}$$

In view of the considerable amount of algebra performed in obtaining Eq. (12), we should check its validity in a situation where the correct result is already known: as $\omega_{p-}/\omega_s \rightarrow 0$. We use the series

$$\sqrt{1 - x} = 1 - \frac{x}{2} - \frac{x^2}{8} - \frac{x^3}{16} - \ldots.$$

$$= 1 - \frac{x}{2} \left(1 + \frac{x}{4} + \frac{x^2}{8} + \ldots. \right) \tag{13}$$

which is valid for $|x| < 1$. With $x \equiv (4\omega_{po}^2 r \cos\gamma/g) \equiv 4(r^2/G^2)(\omega_{po}/\omega_s) \cos\gamma$, we use eqs. (12) and (13) to find

$$\omega_{p-} = \frac{g}{2\omega_{po} r \cos\gamma} \left(\frac{2\omega_{po}^2 r \cos\gamma}{g} \right) \left(1 + \frac{4\omega_{po}^2 r \cos\gamma}{4g} + \frac{16\omega_{po}^4 r^2 \cos^2\gamma}{8g^2} + \ldots. \right)$$

$$= \omega_{po} \left\{ 1 + \frac{\omega_{po}^2}{[g/(r\cos\gamma)]} + \frac{2\omega_{po}^4}{[g/(r\cos\gamma)]^2} + \ldots. \right\} \tag{14}$$

This clearly has the correct limit (ω_{po}) as (ω_{p-}/ω_s) approaches zero. We now use Eq. (12) to evaluate ω_{p-} for the top described in the exercise. The numerical values are r = 4.00 cm, G = 3.00 cm, and ω_s = 40 rad/s. Therefore $\omega_{po} = (980)(4.00)/[(3.00)^2 (40)]$ = 10.89 rad/s. Then Eq. (12) yields

$$\omega_{p-} = \frac{980}{2(10.89)(4.00) \cos\gamma} \left[1 - \sqrt{1 - \frac{4(10.89)^2 (4.00) \cos\gamma}{980}} \right]$$

$$= \frac{11.25}{\cos\gamma} \left(1 - \sqrt{1 - 1.936 \cos\gamma} \right) \tag{15}$$

Equation (15) gives a real value for ω_{p-} only for $\cos\gamma < 0.5165$, or for $\gamma > 58.9^\circ$. We display the values obtained for several angles between 60° and 165°. Notice that for $\gamma = 90^\circ$, $\omega_{p-} = \omega_{po}$, as can be seen from Eq. (14).

γ (°)	ω_{p-} (rad/s)	γ (°)	ω_{p-} (rad/s)
60	18.6	120	9.1
75	12.8	135	8.6
90	10.9	150	8.3
105	9.8	165	8.1

(Angles greater than 90° can be realized with the help of a needle-like support at point O.) The values given here make it obvious that Equation (11) -- that is, Eq. (10-36) -- does not yield an accurate value for ω_{p-} when ω_{po} is comparable ω_s. (In this exercise, $\omega_{po} = 0.27\ \omega_s$.)

10-47

(a) The mass M_1 of top 1 is given by $M_1 = \rho_1 V_1$, while the mass M_2 of top 2 is $M_2 = \rho_2 V_2$. With $\rho_2 = \rho_1$, we have $M_2/M_1 = V_2/V_1$. Since top 2 is λ times as large as top 1 in each linear dimension, we find that $V_2/V_1 = \lambda^3$, so $M_2 = \underline{\lambda^3 M_1}$.

(b) The moment of inertia of a homogeneous object is given by $I = \int r^2 dm = \int r^2 \rho\, dV = \rho \int r^2 dV$. Therefore $I_2 = (\lambda^2)(\lambda^3) I_1 = \underline{\lambda^5 I_1}$.

(c) For $\lambda = 1.2$, we find $M_2 = 1.73 M_1$ and $I_2 = 2.49 I_1$. For $\lambda = 2.0$, we find $M_2 = 8 M_1$ and $I_2 = 32 I_1$. For $\lambda = 5.0$, we find $M_2 = 125 M_1$ and $I_2 = 3125 I_1$.

10-48

(a) The spin angular momentum of each top is given by $L_s = I\omega_s$. Specifically, $L_{sA} = I_A \omega_{sA}$ and $L_{sB} = I_B \omega_{sB}$. With $I_B = \lambda^5 I_A$, as shown in Exercise 10-47, and with $\omega_{sB} = \omega_{sA}$, we find $L_{sB} = \lambda^5 I_A \omega_{sA} = \underline{\lambda^5 L_{sA}}$.

(b) The spin kinetic energy of each top is given by $K_s = \frac{1}{2} I \omega_s^2 = \frac{1}{2} L_s \omega_s$. Therefore $K_{sB} = \frac{1}{2} L_{sB} \omega_{sB} = \frac{1}{2}\lambda^5 L_{sA}\omega_{sB} = \underline{\lambda^5 K_{sA}}$.

(c) According to Eq. (10-36), the precessional angular speed is given by

$$\omega_{pA} = \frac{r_A g}{G_A^2} \cdot \frac{1}{\omega_{sA}}$$

independent of the angle γ_A between $\vec{\omega}_{sA}$ and the upward vertical. Top B is spinning with the same angular speed $\omega_{sB} = \omega_{sA}$. If we <u>assume</u> that Eq. (10-36) also applies to the precession of top B, then we have

$$\omega_{pB} = \frac{r_B g}{G_B^2} \frac{1}{\omega_{sB}}$$

With $r_B = \lambda r_A$, $G_B^2 = \lambda^2 G_A^2$, we find

$$\omega_{pB} = \frac{\lambda}{\lambda^2} \frac{r_A g}{G_A^2} \frac{1}{\omega_{sA}} = \frac{1}{\lambda} \omega_{pA}$$

(d) Using the result of part (c) and $\omega_{sB} = \omega_{sA}$, we find

$$\frac{\omega_{pB}}{\omega_{sB}} = \frac{1}{\lambda} \left(\frac{\omega_{pA}}{\omega_{sA}} \right)$$

(e) If $\omega_{pA}/\omega_{sA} = 1.0 \times 10^{-3}$ and $\lambda = 2.0$, we find $\omega_{pB}/\omega_{sB} = 5.0 \times 10^{-4}$. Since this is much less than unity, we can expect this result to be accurate.

10-49

(a) The moment of inertia I of the entire top is given by $I = I_h + I_c$, where I_h is the moment of inertia of the hemisphere, and I_c is the moment of inertia of the cylinder (or disk). A symmetry argument shows that the moment of inertia of the hemisphere equals one-half the moment of inertia of an entire solid sphere possessing the same mass density. But the mass M of the hemisphere is also equal to one-half the mass M_s of an entire solid sphere of the same mass density. Therefore

$$I_h = \frac{1}{2} \left(\frac{2M_s R^2}{5} \right) = \frac{1}{2} \left[\frac{2(2M)R^2}{5} \right] = \frac{2MR^2}{5}$$

Since the moment of inertia of the disk about the rod is given by $I_c = \frac{1}{2}mc^2$, we have $I = \underline{\frac{2}{5}MR^2 + \frac{1}{2}mc^2}$.

(b) By symmetry, the center of mass of the top lies along the central axis. It lies at a distance r from point F given by

$$r = \frac{Mr_h + mr_c}{(M + m)}$$

where r_h and r_c are the distances from point F to the centers of mass of the hemisphere and of the cylinder, respectively. Using the result of Exercise 9-28, we have $r_h = H + 5R/8$. Referring to Fig. 10E-49, we can see that $r_c = H + R + \ell$. Therefore we find

$$r = \frac{[M(H + 5R/8) + m(H + R + \ell)]}{(M + m)}$$

(c) Under the assumption that $\omega_s \gg \omega_p$, Eq. (10-36) can be applied:

$$\omega_p = \frac{rg}{G^2 \omega_s} = \frac{rg}{(M + m)G^2 \omega_s} = \frac{rg}{I\omega_s}$$

$$= \frac{[M(H + 5R/8) + m(H + R + \ell)]g}{\left[\frac{2}{5} MR^2 + \frac{1}{2} mc^2\right]}$$

$$= \frac{[(H/R + 5/8) + (m/M)(H/R + 1 + \ell/R)]g}{[(2/5) + (mc^2/2MR^2)]R\omega_s}$$

(d) By examining the result given in part (c), or by referring to Fig. 10E-49, we conclude that the most rapid precession will occur for $\ell = \ell_{max} = D - d/2$, and the slowest precession will occur for $\ell = \ell_{min} = d/2$.

(e) Using the results of parts (c) and (d), we find that the desired ratio is given by

$$\frac{\max(\omega_p)}{\min(\omega_p)} = \frac{[(H/R + 5/8) + (m/M)(H/R + 1 + D/R - d/2R)]}{[(H/R + 5/8) + (m/M)(H/R + 1 + d/2R)]}$$

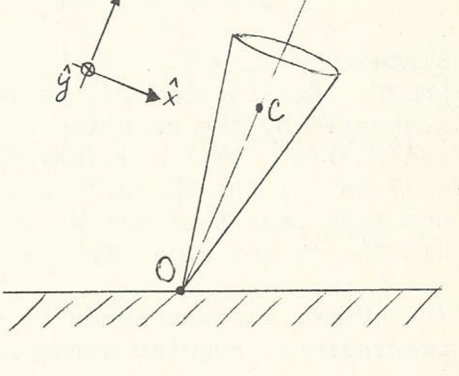
Inserting the given numerical values, we obtain

$$\frac{\max(\omega_p)}{\min(\omega_p)} = \frac{[(0.50 + 5/8) + (0.20)(0.50 + 1 + 3.0 - 0.10)]}{[(0.50 + 5/8) + (0.20)(0.50 + 1 + 0.10)]}$$

$$= \frac{[(1.125) + (0.20)(4.40)]}{[(1.125) + (0.20)(1.60)]}$$

$$= \frac{2.005}{1.445} = \underline{1.39}$$

10-50



(a) For the purposes of determining the location $\vec{r}_c$ of the center of mass, we adopt the coordinate system shown at the right. The origin is at the t p of the cone, and the positive z axis lies along the cone's symmetry axis. By symmetry, $x_c = y_c = 0$. We determine the z coordinate from $z_c = \int z\, dm \div \int dm$, where the integrals are evaluated over the entire volume of the cone. Referring to the figure, we can see that the mass element is given by $dm = \rho A(z)dz = \rho \pi R^2 (z/h)^2 dz$. Therefore we have

$$z_c = \frac{(\rho \pi R^2/h^2) \int_0^h z^3 dz}{(\rho \pi R^2/h^2) \int_0^h z^2 dz}$$

$$= \frac{(h^4/4)}{(h^3/3)} = \frac{3h}{4}$$

as desired.

(b) The moment of inertia of the cone is the sum of the moments of inertia of a set of cylindrical disks, each having a radius $r(z)$ and a mass $dm = \rho \pi r^2(z)dz$. That is,

$$I = \int dI = \int_0^h \frac{1}{2} r^2(z) \rho \pi r^2(z) dz$$

Since $r(z) = Rz/h$, we have

$$I = \frac{\rho \pi R^4}{2h^4} \int_0^h z^4 dz = \frac{\rho \pi R^4}{2h^4} \left(\frac{h^5}{5}\right) = \frac{\rho \pi R^4 h}{10}$$

However, the mass $M = \rho V = \rho R^2 h/3$, so I can be rewritten as $3MR^2/10$, as desired.

(c) Under the assumption that $\omega_p \ll \omega_s$, we apply Eq. (10-36) to obtain

$$\omega_p = \frac{z_c g}{(I/M)\omega_s} = \frac{(3h/4)g}{(3R^2/10)\omega_s} = \frac{5hg}{2R^2 \omega_s}$$

(d) Inserting $h = 10.0$ cm, $R = 3.0$ cm, and $\omega_s = 5800$ rotations per minute = 607.4 rad/s, we find

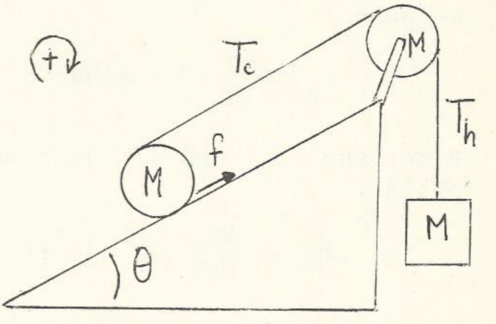
$$\omega_p = \frac{5(10.0)(980)}{2(3.0)^2(607.4)} = 4.48 \text{ rad/s}$$

Since $\omega_p/\omega_s = 7.38 \times 10^{-3}$, the condition $\omega_p \ll \omega_s$ is fulfilled. [NOTE: Examination of the stricter, more complicated criterion suggested by the solution for Exercise 10-46 reveals that $(4d^2/G^2)(\omega_p/\omega_s) = 4(3h/4)^2(\omega_p/\omega_s) \div (3R^2/10) = 4(7.5)^2 \times (7.38 \times 10^{-3}) \div (2.7) = 0.615$, which is less than unity but not much less than unity. We must therefore recognize that Eq. (10-36) is not entirely valid here.]

(e) Under circumstances in which Eq. (10-36) is valid, the precessional angular speed is independent of the tilt angle α.

(When the spin angular momentum greatly exceeds the angular momentum due to motion of the top's center of mass, the magnitude of the torque and the magnitude of the horizontal component of the angular momentum are both proportional to sin α.) However, as pointed out in part (d) above, the top described in this exercise is not spinning fast enough to completely justify Eq. (10-36). The value found in part (d) for ω_p is the correct precession rate for $\alpha = 90^o$, but use of Eq. (12) in the solution for Exercise 10-46 shows that ω_p exceeds $(5hg/2R^2\omega_s)$ for $\alpha < 90^o$, and that $\omega_p < 5hg/2R^2\omega_s$ for $\alpha > 90^o$. Specifically, ω_p ranges from a maximum of 5.53 rad/s for $\alpha = 0^\circ$ to a minimum of 3.95 rad/s for $\alpha = 180^o$. For the angle $\alpha \simeq 25^o$ shown in Fig. 10E-50, we find $\omega_p \simeq 5.38$ rad/s, which is about 20% greater than the value obtained for ω_p in part (d).

10-51



The notation is indicated in the figure. The subscripts h, p, and c refer to hanging weight, pulley, and cylinder, respectively.

(a) We first apply Newton's second law to the hanging object:

$$Mg - T_h = Ma_h \tag{1}$$

Next we apply the rotational form of Newton's law to the pulley:

$$(T_n - T_c)R = I_p\alpha_p \tag{2}$$

Because the string does not slip on the pulley, we also have

$$R\alpha_p = a_h \tag{3}$$

Applying Newton's second law to the cylinder, we find

$$T_c - Mg\sin\theta + f = Ma_c \tag{4}$$

and

$$(T_c - f)R = I_c\alpha_c \tag{5}$$

where f is the frictional force (positive upward). Because there is no slippage between the cylinder and the incline, we also have

$$R\alpha_c = a_c \tag{6}$$

Assuming that the string stays taut, we must have

$$a_c = \frac{a_h}{2} \tag{7}$$

Equations (1) - (7), with $I_p = I_c = \frac{1}{2}MR^2$, can be used to obtain the following equations in the four unknowns T_h, T_c, f, and a_h.

$$-T_h + Mg = Ma_h \quad (8)$$

$$T_h - T_c = \frac{1}{2} Ma_h \quad (9)$$

$$T_c - Mg \sin\theta + f = \frac{1}{2} Ma_h \quad (10)$$

$$T_c - f = \frac{1}{4} Ma_h \quad (11)$$

We solve Eqs. (8) - (11) for the acceleration of the hanging weight:

$$a_h = \frac{4g}{15} (2 - \sin\theta) \quad (12)$$

Equations (12) and (3) determine the angular acceleration of the pulley:

$$\alpha_p = \frac{4g}{15R} (2 - \sin\theta) \quad (13)$$

Using Eqs. (12), (7), and (6), we find the linear and angular accelerations of the cylinder:

$$a_c = \frac{2g}{15} (2 - \sin\theta) \quad (14)$$

$$\alpha_c = \frac{a_c}{R} = \frac{2g}{15R} (2 - \sin\theta) \quad (15)$$

(b) We use Eqs. (8) and (12) of part (a) to obtain the tension T_h in the vertical section of the string:

$$T_h = M(g - a_h) = Mg \left(\frac{15 - 8 + 4 \sin\theta}{15}\right)$$

$$= Mg \left(\frac{7 + 4 \sin\theta}{15}\right) \quad (16)$$

Then Eq. (9) can be used to determine T_c:

$$T_c = T_h - \frac{Ma_h}{2} = Mg \left\{\left(\frac{7 + 4 \sin\theta}{15}\right) - \left[\frac{2}{15} (2 - \sin\theta)\right]\right\}$$

$$= Mg \left(\frac{3 + 6 \sin\theta}{15}\right) = Mg \left(\frac{1 + 2 \sin\theta}{5}\right) \quad (17)$$

We note here that T_h and T_c are both positive, which confirms our earlier assumption that the string remains taut.

(c) From Eqs. (17) and (11), we find that the frictional force is given by

$$f = T_c - \frac{Ma_h}{4} = \frac{Mg}{5}(1 + 2\sin\theta) - \frac{Mg}{15}(2 - \sin\theta)$$

$$= Mg\left(\frac{1 + 7\sin\theta}{15}\right) \qquad (18)$$

Since the normal force N_c between the cylinder and the incline is given by $N_c = Mg\cos\theta$, we must have

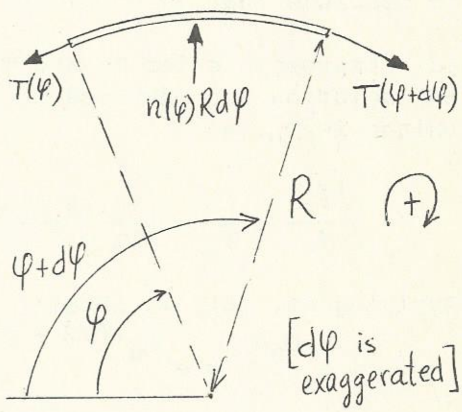
$$\mu_s Mg\cos\theta \geq Mg\left(\frac{1 + 7\sin\theta}{15}\right)$$

Otherwise, there would be slippage. In terms of the coefficient of static friction μ_s, we must have

$$\mu_s \geq \frac{1 + 7\sin\theta}{15\cos\theta} \qquad (19)$$

<u>10-52</u>

We begin by observing that the coefficient of static friction here bears no necessary relationship to the coefficient of static friction described in part (c) of Exercise 10-51.



(a) Referring to the figure at the right, we let $n(\varphi)$ be the normal force per unit length at angle φ. Since the piece of string shown has negligible mass, we must have

$$n(\varphi) = \frac{T(\varphi)}{R} \qquad (1)$$

Furthermore, the rate of change of tension with distance along the string is limited:

$$\left|\frac{dT}{ds}\right| = \frac{1}{R}\left|\frac{dT}{d\varphi}\right| \leq \mu_s n(\varphi) \qquad (2)$$

Using (1) and (2), we find

$$\left|\frac{dT(\varphi)}{ds}\right| \leq \frac{\mu_s T(\varphi)}{R} \qquad (3)$$

(b) Since $|ds| = R|d\varphi|$, Eq. (3) shows that

$$\left|\frac{dT}{d\varphi}\right| \leq \mu_s T(\varphi) \qquad (4)$$

Integrating Eq. (4), we find

$$\left|\ln\left[\frac{T(\varphi_2)}{T(\varphi_1)}\right]\right| \leq \mu_s |(\varphi_2 - \varphi_1)| \qquad (5)$$

Setting $\alpha = |\varphi_2 - \varphi_1|$, $T_> = \max(T_1, T_2)$ and $T_< = \min(T_1, T_2)$, we find

$$\frac{T_>}{T_<} \leq e^{\mu_s \alpha} \qquad (6)$$

as desired.

(c) Referring to Fig. 10E-51, the pulley radii for which the string breaks contact with the pulley are at right angles to the two straight sections. From this observation it is not difficult to conclude that $\alpha = \pi/2 + \theta$.

(d) Expressions for T_h and T_c are given as Eqs. (16) and (17) in the solution for Exercise 10-51. The larger of the two tension values is T_h, so

$$\frac{T_>}{T_<} = \frac{T_h}{T_c} = \frac{7 + 4\sin\theta}{3(1 + 2\sin\theta)}$$

Applying Eq. (6), we obtain

$$e^{\mu_s \alpha} = e^{\mu_s(\pi/2 + \theta)} \geq \frac{7 + 4\sin\theta}{3(1 + 2\sin\theta)} \qquad (7)$$

or

$$\mu_s \geq \frac{1}{(\pi/2 + \theta)} \ln\left[\frac{(7 + 4\sin\theta)}{3(1 + 2\sin\theta)}\right]$$

10-53

We must establish that the momentum $\vec{P}$ of a system of particles satisfies the equation

$$\frac{d\vec{P}}{dt} = \vec{F} \qquad (1)$$

where $\vec{F}$ is the vector sum of all external forces acting on the system. Assume there are N particles, with particle i having mass m_i. We let $\vec{F}_{ij}$ denote the force exerted on particle i by particle j, and we let $\vec{F}_{i,ext}$ denote the sum of the external forces acting on the i^{th} particle. Then for each particle ($1 \leq i \leq N$), Newton's second law implies that

$$m_i \frac{d\vec{v}_i}{dt} = \left(\sum_{\substack{j=1 \\ j \neq i}}^{N} \vec{F}_{ij} \right) + \vec{F}_{i,ext} \qquad (2)$$

The term with $j = i$ is excluded since no particle exerts a force on itself. By summing the Newton's law equation over all the particles, we obtain

$$\sum_{i=1}^{N} m_i \frac{d\vec{v}_i}{dt} = \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \vec{F}_{ij} + \sum_{i=1}^{N} \vec{F}_{i,ext} \qquad (3)$$

Since $\vec{F}_{ij} = -\vec{F}_{ji}$, the first term on the right hand side of Eq. (3) consists of $N(N - 1)/2$ force pairs; each of these pairs has a zero sum. Therefore

$$\sum_{i=1}^{N} m_i \frac{d\vec{v}_i}{dt} = \sum_{i=1}^{N} \vec{F}_{i,ext} \equiv \vec{F} \qquad (4)$$

where $\vec{F}$ is the vector sum of all external forces acting on particles in the system. We must now rewrite the left hand side of the equation. The center of mass location $\vec{r}_c$ satisfies the equation

$$\vec{r}_c = \sum_{i=1}^{N} m_i \vec{r}_i \Big/ \sum_{i=1}^{N} m_i \qquad (5)$$

We differentiate Eq. (5) to show that

$$\vec{P} \equiv \left(\sum_{i=1}^{N} m_i \right) \vec{v}_c = \sum_{i=1}^{N} m_i \vec{v}_i \qquad (6)$$

It is easy to see that Eqs. (6) and (4) imply that

$$\frac{d\vec{P}}{dt} = \vec{F} \qquad (1)$$

as desired.

CHAPTER ELEVEN

Note: In presenting these solutions, we have tried to be consistent in using $G = 6.67 \times 10^{-11}$ N·m^2/kg, $M_e = 5.99 \times 10^{24}$ kg, $R_e = 6.37 \times 10^3$ km. Small discrepancies between your results and the answers given here may be due to use of differing values for G, the earth's mass, the earth's radius, and other quantities.