

$v = \sqrt{2gy}$. Each element of the water stream follows the trajectory of a particle launched horizontally at the same speed and elevation. The "fall time" t satisfies the equation $\frac{1}{2}gt^2 = h - y$, so that $t = \sqrt{2(h - y)/g}$. The horizontal distance traveled during the fall is therefore given by

$$x = vt = \sqrt{2gy} \cdot \sqrt{\frac{2(h - y)}{g}} = 2\sqrt{y(h - y)}$$

(b) If a second hole is located a distance y' below the water surface, the exit stream will have a range $x' = 2\sqrt{y'(h - y')}$. The ranges x' and x are equal for $y' = y$ (a trivial solution) and for $y' = h - y$. That is, if one hole is located a distance y below the water surface, another hole will provide a stream with the same range if it is located a distance $y' = h - y$ below the water surface, or a distance $h - y' = y$ above the bottom of the can. Two holes have the same range if one is as far above the bottom of the can as the other is below the water surface.

(c) For a given water depth h , the y value which provides the maximum range can be determined by setting $dx/dy = 0$. The derivative is given by

$$\frac{dx}{dy} = \frac{d}{dy} \left[2\sqrt{y(h - y)} \right] = \frac{2(h - 2y)}{\sqrt{y(h - y)}}$$

This vanishes for $y \equiv y^* = h/2$. (It is easily shown that the corresponding range is a maximum and not a minimum. The function $\sqrt{y(h - y)}$ is nonnegative and exhibits only one extremum in the range $0 \leq y \leq h$. Since $x = 0$ for $y = 0$ and for $y = h$, the extremum of x at $y \equiv y^* = h/2$ must be a maximum.)

CHAPTER SEVENTEEN

17-9

The initial diameter of the steel rim is d ; it must be heated in order to increase its diameter to d' . The corresponding circumferences are $l = \pi d$ and $l' = \pi d'$. Denoting the coefficient of linear expansion by α , the required temperature increase ΔT can be found from the equation $l' = l(1 + \alpha \Delta T)$. Solving for ΔT , we obtain

$$\Delta T = \frac{l' - l}{\alpha l} = \frac{d' - d}{\alpha d}$$

Using the numerical values $d = 0.998$ m, $d' = 1.000$ m, and $\alpha = 10.5 \times 10^{-6} \text{ K}^{-1}$ (from Table 17-1), we find

$$\Delta T = (0.002)/[(10.5 \times 10^{-6})(0.998)] = 191 \text{ K} = \underline{191 \text{ }^\circ\text{C}}.$$

17-10

The heat requirement is evidently the sum of five contributions. The first stage is the warming of the ice from $-10 \text{ }^\circ\text{C}$ to the melting point ($0 \text{ }^\circ\text{C}$). According to Table 17-3, the specific heat capacity of ice is $0.502 \text{ cal}/(\text{g}\cdot^\circ\text{C})$. Therefore the heat required in the first stage is given by $\Delta H_1 = mc_1 \Delta t_1 = (1.00 \text{ g})[(0.502 \text{ cal}/(\text{g}\cdot^\circ\text{C})) \times (10 \text{ }^\circ\text{C})] = \underline{5.02 \text{ cal}}$. The second stage is the melting of the ice at $0 \text{ }^\circ\text{C}$ and 1 atmosphere of pressure. According to Table 17-4, the latent heat for the melting of ice is 79.7 cal/g , so $\Delta H_2 = mL_2 = (1.00 \text{ g})(79.7 \text{ cal/g}) = \underline{79.7 \text{ cal}}$. The third stage is the heating of the water from $0 \text{ }^\circ\text{C}$ to $100 \text{ }^\circ\text{C}$, the boiling point. Since the specific heat capacity of water is approximately constant [at $1.000 \text{ cal}/(\text{g}\cdot^\circ\text{C})$], the heat required is given by $\Delta H_3 = mc_3 \Delta t_3 = (1.00 \text{ g})[(1.000 \text{ cal}/(\text{g}\cdot^\circ\text{C}))(100 \text{ }^\circ\text{C})] = \underline{100 \text{ cal}}$. The fourth stage is the boiling of the water at a temperature of $100 \text{ }^\circ\text{C}$ and at a constant pressure of 1.00 atm . According to Table 17-4, the latent heat for the boiling of water at 1.00 atm is 539.6 cal/g , so $\Delta H_4 = mL_4 = (1.00 \text{ g})(539.6 \text{ cal/g}) = \underline{539.6 \text{ cal}}$. The fifth and final stage is the heating of the steam from $100 \text{ }^\circ\text{C}$ to $120 \text{ }^\circ\text{C}$ (at a constant pressure of 1.00 atm). Assuming that between $100 \text{ }^\circ\text{C}$ and $120 \text{ }^\circ\text{C}$ the specific heat capacity of steam is constant and has the value $0.481 \text{ cal}/(\text{g}\cdot^\circ\text{C})$ given in Table 17-3, we find $\Delta H_5 = mc_5 \Delta t_5 = (1 \text{ g})[0.481 \text{ cal}/(\text{g}\cdot^\circ\text{C}))(20 \text{ }^\circ\text{C})] = \underline{9.62 \text{ cal}}$. The total heat requirement $\Delta H = \Delta H_1 + \Delta H_2 + \Delta H_3 + \Delta H_4 + \Delta H_5 = (5.02 + 79.7 + 100 + 539.6 + 9.62) = 733.9 = \underline{734 \text{ cal}} = \underline{0.734 \text{ kcal}}$.

17-11

Clearly it is intended that this exercise be solved by assuming that in the final state, the system will consist of $100 \text{ }^\circ\text{C}$ water (with a $100 \text{ }^\circ\text{C}$ horseshoe immersed in it), plus some quantity of steam at $100 \text{ }^\circ\text{C}$. (It is easy to believe that the heat transfer from the hot horseshoe to its surroundings will occur in such a way that "portions" of the water will heat up more rapidly than others, and that the final state will involve a somewhat lower temperature than $100 \text{ }^\circ\text{C}$ for the liquid. Of course, there is also the matter of heat transfer between the water bath and its surroundings. It is not possible to accommodate these complications without additional information.) The cooling of 2.0 kg of iron from $1200 \text{ }^\circ\text{C}$ to $100 \text{ }^\circ\text{C}$ releases $m_{\text{Fe}}c_{\text{Fe}} \Delta t_{\text{Fe}} = (2.0 \text{ kg}) \times [0.108 \text{ kcal}/(\text{kg } ^\circ\text{C})](1100 \text{ }^\circ\text{C}) = 237.6 \text{ kcal}$. The heating of 8.0 kg of water from $50 \text{ }^\circ\text{C}$ to $100 \text{ }^\circ\text{C}$ requires $m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}} \Delta t_{\text{H}_2\text{O}} = (8.0 \text{ kg})[(1.000 \text{ kcal}/(\text{kg } ^\circ\text{C}))(50 \text{ }^\circ\text{C})] = 40 \text{ kcal}$. Therefore there are $237.6 - 40 = 197.6 \text{ kcal}$ of heat available to produce steam. The mass of steam produced equals 197.6 kcal , divided by the latent heat of boiling for water (which is 539.6 kcal/kg). The quotient is 0.37 kg .

17-13

Assuming (as is intended) that the tube is turned fairly quickly, the lead shot falls through a distance $h = 1.0$ m each time the tube is turned. In $N = 15$ turns of the tube, the total energy which is converted from potential energy into kinetic energy and then into heat is given by $Nmgh$, where m is the mass of the lead shot. Assuming that all of the heat remains in the lead shot, the temperature rise $\Delta t = 1.0$ °C of the shot is related to the mechanical energy release by

$$mc\Delta t = Nmgh$$

where c is the specific heat capacity of lead. From Table 17-3, we have $c = 0.0306$ kcal/(kg·°C). Therefore we have

$$\begin{aligned} 1 &= \frac{Ngh}{c\Delta t} = \frac{(15)(9.80 \text{ m/s}^2)(1.0 \text{ m})}{[(0.0306 \text{ kcal}/(\text{kg}\cdot^\circ\text{C}))](1^\circ\text{C})} \\ &= 4.8 \times 10^3 \text{ J/kcal} \end{aligned}$$

According to this experiment, 4.8×10^3 J of mechanical energy is equivalent to 1 kcal of heat. The main source of error in the experiment is the fact that not all of the heat generated is within the lead shot. During the inelastic impacts of the shot with the stoppers at the ends of the tube, some of the energy is deposited in the stoppers rather than in the shot. Furthermore, some of the heat generated within the lead shot is transferred to the slightly cooler surroundings. Both of these effects tend to result in an overestimate for the mechanical equivalent of heat. An additional source of uncertainty is the difficulty of accurately measuring a slight temperature rise in a small amount of a solid.

17-15

(a) Since the pressure and the mass of the gas sample remain constant, Charles' law is applicable:

$$\frac{V_i}{T_i} = \frac{V_f}{T_f} \quad (1)$$

The cylindrical chamber has a constant cross-sectional area A , so that

$$\frac{V_f}{V_i} = \frac{Ah_f}{Ah_i} = \frac{h_f}{h_i} \quad (2)$$

Combining eqs. (1) and (2), the final height is given by

$$h_f = h_i \left(\frac{V_f}{V_i} \right) = h_i \left(\frac{T_f}{T_i} \right)$$

With $t_i = 20$ °C and $t_f = 100$ °C, we have $T_i = 293.15$ K and $T_f = 373.15$ K. Using $h_i = 4.00$ cm, we find

$$\begin{aligned} h_f &= (4.00 \text{ cm}) \left(\frac{373.15 \text{ K}}{293.15 \text{ K}} \right) \\ &= \underline{5.09 \text{ cm}} \end{aligned}$$

(b) Since there is a fixed quantity of gas in the cylinder, the quantity pV/T is constant; since V is held constant, the final pressure is given by

$$P_f = P_i \left(\frac{T_f}{T_i} \right) \quad (1)$$

In terms of the ambient exterior air pressure (which we take to be $p_{\text{atm}} = 1.013 \times 10^5$ Pa), the initial and final pressures in the cylinder are

$$p_i = [p_{\text{atm}} + (m_i g/A)] \quad (2)$$

and

$$\begin{aligned} p_f &= [p_{\text{atm}} + (m_f g/A)] \\ &= [p_{\text{atm}} + (m_i g/A) + (\Delta m g/A)] \quad (3) \end{aligned}$$

Solving eqs. (1) - (3) for the necessary additional mass Δm , we find

$$\begin{aligned} \Delta m &= (A/g) (p_f - p_i) \\ &= (A/g) p_i \left[\left(\frac{T_f}{T_i} \right) - 1 \right] \\ &= (A/g) [p_{\text{atm}} + (m_i g/A)] \left[\left(\frac{T_f}{T_i} \right) - 1 \right] \quad (4) \end{aligned}$$

The cross-sectional area $A = (\pi/4) (4.00 \text{ cm})^2 = 12.57 \text{ cm}^2 = 1.257 \times 10^{-3} \text{ m}^2$. Inserting $p_{\text{atm}} = 1.013 \times 10^5$ Pa, $m_i = 13.0$ kg, $g = 9.80 \text{ m/s}^2$, and the given temperatures, we obtain

$$\begin{aligned} \Delta m &= \left(\frac{1.257 \times 10^{-3}}{9.80} \right) \left[1.013 \times 10^5 + \frac{(13.0)(9.80)}{(1.257 \times 10^{-3})} \right] \\ &\times \left(\frac{373.15}{293.15} - 1 \right) \\ &= \left[\frac{(1.257 \times 10^{-3})(1.013 \times 10^5)}{(9.80)} + 13.0 \right] (0.2729) \end{aligned}$$

That is, $\Delta m = (12.99 + 13.0)(0.2729) = \underline{7.09 \text{ kg}}$.

17-16

(a) In relatively sparse air, there will be less lift on the wings of the airplane. Other factors being equal, take-off runs will be longer, climbing rates will be smaller, and the descent during landing approaches will be more rapid in the less dense air at high elevations and/or on warm days.

(b) The density ρ is inversely proportional to the volume of a given mass of gas, so that

$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2}$$

Because the pressures are assumed to be equal, Charles' law is applicable. Writing it in terms of the densities, we have

$$\frac{\rho_2}{\rho_1} = \frac{T_1}{T_2}$$

With $T_2 = 273.15 \text{ K}$ ($t_2 = 0 \text{ }^\circ\text{C}$) and $T_1 = 303.15 \text{ K}$ ($t_1 = 30 \text{ }^\circ\text{C}$), we find

$$\frac{\rho_2}{\rho_1} = \frac{303.15}{273.15} = 1.11$$

That is, at $0 \text{ }^\circ\text{C}$ the density is 11% higher than it is at $30 \text{ }^\circ\text{C}$.

(c) We are given the hydrostatic pressure profile for an isothermal atmosphere, which indicates that we are expected to assume that the pressure at Stapleton Field can be found from that profile with sufficient accuracy. Once the pressure is determined, we will find the density corresponding to that pressure and the given (higher) temperature. [NOTE: There is no claim that the method suggested in the exercise statement uses the only sensible set of assumptions, or even the most reasonable set, meteorologically speaking. It is possible to question (1) the use of an isothermal profile, rather than an "adiabatic profile" (see Chapter 19), and even (2) the assumption of complete hydrostatic equilibrium over such large distances. However, the assumptions used in this exercise do lead to a relatively simple procedure for obtaining a reasonable idea of typical atmospheric density variations at the earth's surface.] We begin by determining the pressure at Stapleton Field with the help of the hydrostatic isothermal profile. Using the subscripts ℓ and s to refer to "Logan" and "Stapleton", we have

$$p_s = p_\ell e^{-[(z_s - z_\ell)/(8150 \text{ m})]}$$

We now determine the density which corresponds to p_s and the given temperature T_s . If the temperatures were equal ($T_s = T_l$), the density ratio ρ_s/ρ_l would equal the pressure ratio p_s/p_l (by Boyle's law). Since $T_s \neq T_l$, we must use the more general relationship $(p/\rho T) = \text{constant}$ (since the gas composition is assumed to be the same at the two locations). That is,

$$\begin{aligned} \frac{\rho_s}{\rho_l} &= \frac{T_l}{T_s} \frac{p_s}{p_l} \\ &= \frac{T_l}{T_s} e^{-[(z_s - z_l)/(8150 \text{ m})]} \end{aligned}$$

With $T_l = 0^\circ \text{C} = 273.15 \text{ K}$, $T_s = 30^\circ \text{C} = 303.15 \text{ K}$, and $z_s - z_l = 1600 \text{ m}$, we obtain

$$\begin{aligned} \frac{\rho_s}{\rho_l} &= \left(\frac{273.15}{303.15}\right) e^{-(1600/8150)} \\ &= (0.9010) e^{-0.1963} \\ &= 0.74 \end{aligned}$$

Under the given conditions and assumptions, the density is 26% lower at Stapleton Field than at Logan Airport. Equivalently, the density at Logan Airport is 35% higher than at Stapleton Field.

17-17

The total pressure in the tank has been reduced from 31 atm to 26 atm. At the latter pressure, the gas originally in the tank would occupy $31/26 \text{ m}^3$. Since 1.00 m^3 remains in the tank, the amount of gas used was $5/26 \text{ m}^3$ at 26 atm. At the same temperature and at atmospheric pressure, this would occupy a volume 26 times as large: 5.00 m^3 .

17-18

(a) In terms of the given quantities, the number of kilomoles of sample x is given by $n_x = p_x V_x / RT_x$; similarly, $n_y = p_y V_y / RT_y$. After the partition is removed, the number of kilomoles $n_f = n_x + n_y$. Applying the ideal gas law to the entire sample, and using $V_f = V_x + V_y$, we have

$$p_f = \frac{n_f RT_f}{V_f} = \frac{RT_f (n_x + n_y)}{V_x + V_y}$$

That is,

$$p_f = p_x \left(\frac{T_f}{T_x} \right) \left(\frac{V_x}{V_x + V_y} \right) + p_y \left(\frac{T_f}{T_y} \right) \left(\frac{V_y}{V_x + V_y} \right)$$

(b) With $V_f = V_x + V_y + V_p$, the result is

$$\begin{aligned} p_f &= \frac{n_f RT_f}{V_f} = \frac{RT_f (n_x + n_y)}{V_x + V_y + V_p} \\ &= p_x \left(\frac{T_f}{T_x} \right) \left(\frac{V_x}{V_x + V_y + V_p} \right) + p_y \left(\frac{T_f}{T_y} \right) \left(\frac{V_y}{V_x + V_y + V_p} \right) \end{aligned}$$

17-19

We denote the true ambient pressure by p_a , the initial pressure of the trapped air by p_t , and the initial barometer reading by p_r . Hydrostatic equilibrium of the mercury at the surface of the reservoir implies that

$$p_a = p_t + p_r \quad (1)$$

When the open end of the barometer is lowered further into the reservoir, the pressure of the trapped air increases to p'_t , while the barometer reading decreases to p'_r . The total pressure is unchanged:

$$p_a = p'_t + p'_r \quad (2)$$

The volume of the trapped air decreases from V_t to $V'_t = 2V_t/3$, so Boyle's law implies that

$$p'_t = 3p_t/2 \quad (3)$$

From eqs. (1) - (3), we find that

$$\begin{aligned} p_t &= 2(p_r - p'_r) \\ &= 2(740 - 730) \text{ mm Hg} \\ &= 20 \text{ mm Hg} \end{aligned}$$

Then eq. (1) implies that $p_a = 740 + 20 = \underline{760 \text{ mm Hg}}$.

17-20

We let n_1 and n_2 denote the number of moles of gas in the large

and small bulbs, in the final configuration. Denoting the final temperatures by T_1 and T_2 and the final pressure by p_f , the ideal gas law implies that

$$p_f V_1 = n_1 R T_1 \quad (1)$$

and

$$p_f V_2 = n_2 R T_2 \quad (2)$$

where $V_1 = 400 \text{ cm}^3$ and $V_2 = 200 \text{ cm}^3$. We also have

$$p_o V_o = n R T_o \quad (3)$$

where p_o , $V_o = V_1 + V_2$, and T_o are the initial pressure, volume, and temperature. Using eqs. (1) - (3) in the equation $n_1 + n_2 = n$, we find

$$\frac{p_f V_1}{R T_1} + \frac{p_f V_2}{R T_2} = \frac{p_o V_o}{R T_o}$$

Solving for p_f , we obtain

$$p_f = \frac{p_o V_o}{T_o \left(\frac{V_1}{T_1} + \frac{V_2}{T_2} \right)}$$

Inserting the numerical values $p_o = 1.00 \text{ atm}$, $T_o = 293.15 \text{ K}$ ($t_o = 20 \text{ }^\circ\text{C}$), $T_1 = 373.15 \text{ K}$ ($t_1 = 100 \text{ }^\circ\text{C}$), and $T_2 = 273.15 \text{ K}$ ($t_2 = 0 \text{ }^\circ\text{C}$), we find

$$p_f = \frac{(1.00)(600)}{(293.15) \left[\left(\frac{400}{373.15} \right) + \left(\frac{200}{273.15} \right) \right]}$$

$$= \underline{1.13 \text{ atm}}$$

17-21

If the expansion of the bulb is neglected, the change Δl_1 in the length of the mercury column is given by

$$\Delta l_1 = \frac{\gamma_{\text{Hg}} V_{\text{Hg}} \Delta t}{A} \quad (1)$$

where γ_{Hg} is the coefficient of volume expansion of mercury, V_{Hg} is the volume of mercury in the bulb, and A is the cross-sectional area of the capillary. However, the bulb's interior volume will increase by an amount $3\alpha_g V$, where α_g is the coefficient of linear expansion of glass and V is the volume of the bulb.

Therefore the change in length of the mercury column is not Δl_1 , but the smaller amount Δl_2 given by

$$\begin{aligned}\Delta l_2 &= \frac{(\gamma_{\text{Hg}} v_{\text{Hg}} - 3\alpha_g v)\Delta t}{A} \\ &= (\gamma_{\text{Hg}} - 3\alpha_g) \frac{v\Delta t}{A}\end{aligned}\quad (2)$$

where we have used the fact that $v_{\text{Hg}} = v$. From eqs. (1) and (2), we find

$$\frac{\Delta l_1 - \Delta l_2}{\Delta l_1} = \frac{\gamma_{\text{Hg}} - (\gamma_{\text{Hg}} - 3\alpha_g)}{\gamma_{\text{Hg}}} = \frac{3\alpha_g}{\gamma_{\text{Hg}}}\quad (3)$$

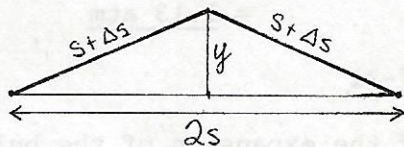
Referring to Tables 17-1 and 17-2, we find $\alpha_g = 8.5 \times 10^{-6} \text{ K}^{-1}$ and $\gamma_{\text{Hg}} = 181.9 \times 10^{-6} \text{ K}^{-1}$, so that $(\Delta l_1 - \Delta l_2)/\Delta l_1 = \underline{0.14}$.

17-22

The height of the column is given in terms of the pressure p_0 and the density ρ of the barometric liquid by $h = p_0/\rho g$, where g is the acceleration due to gravity. As a consequence of a process in which the temperature rises by ΔT but the pressure does not change, the fractional change in the height of the column is given by $\Delta h/h = -\Delta\rho/\rho$, where $\Delta\rho$ is the density change due to thermal expansion. Letting V be the mass of a fixed mass M the liquid, we have $\rho V = M$, so that $\Delta(\rho V) = 0$. This implies that $-\Delta\rho/\rho = \Delta V/V = \gamma\Delta T$. Therefore $\Delta h/h = \gamma\Delta T$, or $\underline{\Delta h = \gamma h\Delta T}$, as desired.

17-23

As indicated in the figure at the right, we let the initial length be $2s$ and the final total length be $2(s + \Delta s)$. The height of the center of the buckled rail is denoted by y . Assuming that the standard coefficient α of linear expansion can be used (in spite of the fact that the ends are anchored), we have $\Delta s = \alpha s\Delta T$.



By the Pythagorean theorem, $y = \sqrt{(s + \Delta s)^2 - s^2} = \sqrt{2s\Delta s + (\Delta s)^2} = s\sqrt{2\alpha\Delta T + (\alpha\Delta T)^2}$. With $s = 15.0 \text{ m}$, $\alpha = 10.5 \times 10^{-6} \text{ K}^{-1}$, and $\Delta T = 50 \text{ K}$, we obtain $y = (15.0 \text{ m})\sqrt{10.5 \times 10^{-4} + (5.03 \times 10^{-4})^2} = \underline{0.49 \text{ m}}$.

17-24

The heat gained by the can and the ice is equal to the heat lost

by the hot water. Presuming that the hot water is sufficient in quantity to melt all of the ice, the heat gained by the can and the ice is given by

$$\Delta H_{\text{can \& ice}} = m_{\text{ice}} L_{\text{ice}} + (m_{\text{ice}} c_{\text{H}_2\text{O}} + m_{\text{can}} c_{\text{Al}}) (t_f - t_c)$$

Here m_{ice} is the mass of ice, m_{can} is the mass of the can, t_c is the initial temperature of the can and ice, t_f is the final temperature, L_{ice} is the latent heat of the melting of ice, $c_{\text{H}_2\text{O}}$ is the specific heat capacity of water, and c_{Al} is the specific heat capacity of aluminum. The heat lost by the hot water is

$$-\Delta H_{\text{hot water}} = m_{\text{hot water}} c_{\text{H}_2\text{O}} (t_h - t_f)$$

where $m_{\text{hot water}}$ is the mass of the hot water and t_h is its initial temperature. Setting $\Delta H_{\text{can \& ice}} = -\Delta H_{\text{hot water}}$, we have

$$\begin{aligned} m_{\text{ice}} L_{\text{ice}} + (m_{\text{ice}} c_{\text{H}_2\text{O}} + m_{\text{can}} c_{\text{Al}}) t_f - (m_{\text{ice}} c_{\text{H}_2\text{O}} + m_{\text{can}} c_{\text{Al}}) t_c \\ = m_{\text{hot water}} c_{\text{H}_2\text{O}} t_h - m_{\text{hot water}} c_{\text{H}_2\text{O}} t_f \end{aligned}$$

Solving for t_f , we find

$$t_f = \frac{m_{\text{hot water}} c_{\text{H}_2\text{O}} t_h + (m_{\text{ice}} c_{\text{H}_2\text{O}} + m_{\text{can}} c_{\text{Al}}) t_c - m_{\text{ice}} L_{\text{ice}}}{(m_{\text{hot water}} c_{\text{H}_2\text{O}} + m_{\text{ice}} c_{\text{H}_2\text{O}} + m_{\text{can}} c_{\text{Al}})}$$

Inserting the given numerical values, and using $c_{\text{H}_2\text{O}} = 1.00 \text{ cal}/(\text{g}\cdot^\circ\text{C})$, $c_{\text{Al}} = 0.214 \text{ cal}/(\text{g}\cdot^\circ\text{C})$, and $L_{\text{ice}} = 79.7 \text{ cal/g}$, we obtain

$$\begin{aligned} t_f &= \frac{6000 + (480 + 0.428)(0) - (3825.6)}{(75.0 + 48.0 + 0.428)} \\ &= \underline{17.6} \text{ }^\circ\text{C} \end{aligned}$$

(NOTE: If a negative value had been obtained for t_f , it would have shown the incorrectness of our a priori assumption that the hot water was sufficient to melt all of the ice.)

17-25

The nail acquired the temperature t of the flame. When the nail was plunged into the cold water, it lost an amount of heat equal to the heat gained by the water. Taking the specific heat capacity

of iron to be $0.108 \text{ cal}/(\text{g}\cdot^{\circ}\text{C})$, as given in Exercise 17.11, we have

$$\begin{aligned} & (10 \text{ g}) [0.108 \text{ cal}/(\text{g}\cdot^{\circ}\text{C})] (t - 20^{\circ}\text{C}) \\ &= (100 \text{ g}) [1.00 \text{ cal}/(\text{g}\cdot^{\circ}\text{C})] (20^{\circ}\text{C} - 10^{\circ}\text{C}) \end{aligned}$$

Therefore $t - 20^{\circ}\text{C} = (1000/1.08) = 925.9^{\circ}\text{C}$, so that the result for the flame temperature is 946°C .

17-26

(a) We let m_o be the initial mass of water, m_{Fe} be the mass of the iron can, and Δm be the mass of ice added. We let t_h be the initial temperature of the water and the can, $t_c = 0^{\circ}\text{C}$ be the initial temperature of the ice, and t_f be the common final temperature. The law of mixtures (heat gained = heat lost) takes the form

$$L\Delta m + \Delta mc_{\text{H}_2\text{O}} t_f = (m_o c_{\text{H}_2\text{O}} + m_{\text{Fe}} c_{\text{Fe}}) (t_h - t_f)$$

where L is the latent heat of fusion (melting) of ice. Solving for L , we have

$$L = \frac{(m_o c_{\text{H}_2\text{O}} + m_{\text{Fe}} c_{\text{Fe}}) (t_h - t_f) - \Delta mc_{\text{H}_2\text{O}} t_f}{\Delta m}$$

Using $c_{\text{Fe}} = 0.108 \text{ cal}/(\text{g}\cdot^{\circ}\text{C})$, $c_{\text{H}_2\text{O}} = 1.00 \text{ cal}/(\text{g}\cdot^{\circ}\text{C})$, and the given numerical values, we obtain

$$\begin{aligned} L &= \frac{(200.0 + 21.6)(20.0) - (50.0)(1.00)(10.0)}{(50.0)} \\ &= 78.6 \text{ cal/g} = \underline{78.6 \text{ kcal/kg}} \end{aligned}$$

(b) During the first part of the experiment, heat escapes from the warm water to the room. This source of error can be offset by heat gained from the room if the water temperature is lower than the room temperature during the last part of the experiment. By conducting an experiment in which the final temperature is as much below room temperature as the initial temperature is above room temperature (and in which the ice is added steadily, so that the water temperature drops at a uniform rate), it can reasonably be hoped that the competing errors will (almost exactly) cancel.

17-27

We presume that the final temperature is greater than 0°C and less than 100°C , so that water is entirely in the liquid state. Using a notation that is (hopefully) self-evident, the heat gained by the ice is given by

$$\Delta H_{\text{ice}} = m_{\text{ice}} c_{\text{ice}} \Delta t_{\text{ice}} + m_{\text{ice}} L_{\text{ice}} + m_{\text{ice}} c_{\text{water}} \Delta t_{\text{melted ice}} \quad (1)$$

Using $c_{\text{ice}} = 0.502 \text{ cal}/(\text{g}\cdot^{\circ}\text{C})$, $c_{\text{water}} = 1.00 \text{ cal}/(\text{g}\cdot^{\circ}\text{C})$, $L_{\text{ice}} = 79.7 \text{ cal/g}$, and the given numerical values, we find

$$\begin{aligned} \Delta H_{\text{ice}} &= (50.0)(0.502)(40) + (50.0)(79.7) + (50.0)(1.00)t_f \\ &= 4989 \text{ cal} + (50.0 \text{ cal}/^{\circ}\text{C})t_f \end{aligned} \quad (2)$$

where t_f is the final temperature. The heat lost by the steam is given by

$$\begin{aligned} -(\Delta H_{\text{steam}}) &= -m_{\text{steam}} c_{\text{steam}} \Delta t_{\text{steam}} + m_{\text{steam}} L_{\text{steam}} \\ &\quad - m_{\text{steam}} c_{\text{water}} \Delta t_{\text{condensed steam}} \end{aligned} \quad (3)$$

Using $c_{\text{steam}} = 0.481 \text{ cal}/(\text{g}\cdot^{\circ}\text{C})$, $L_{\text{steam}} = 539.6 \text{ cal/g}$, and the given numerical values, we find

$$\begin{aligned} -(\Delta H_{\text{steam}}) &= (11.0)(0.481)(20) + (11.0)(539.6) \\ &\quad + (11.0)(1.00)(100 - t_f) \\ &= 7141.42 \text{ cal} - (11.0 \text{ cal}/^{\circ}\text{C})t_f \end{aligned} \quad (4)$$

Solving eqs. (2) and (4) for t_f , we find $t_f = (2152.42)/61 = \underline{35.3^{\circ}\text{C}}$. (NOTE: A final result colder than 0°C or hotter than 100°C , would have indicated the incorrectness of our a priori assumption that the final state involved only liquid water.)

17-28

(a) Applying the ideal gas law, we have $p_b = n_b RT_0/V_b$.

(b) The ideal gas law implies that $p_c = n_c RT_0/V_c$.

(c) The lid would lift if the net upward pressure force $(p_b - p_c)A_l$ exceeded the weight $M_l g$. The fact that the lid is closed therefore implies that $p_b - p_c \leq M_l g/A_l$.

(d) The lid will lift when $p_b - p_c = M_l g/A_l$. Using the results of parts (a) and (b), this occurs for

$$\frac{n_b RT_l}{V_b} - \frac{n_c RT_l}{V_c} = \frac{M_l g}{A_l}$$

or when

$$T_{\ell} = \frac{M_{\ell} g}{RA_{\ell} [(n_b/V_b) - (n_c/V_c)]}$$

(e) As the system is heated, gas escapes from the box to the surrounding chamber in amounts just sufficient to maintain an internal pressure excess of $M_{\ell} g/A_{\ell}$. Once the peak temperature T' is reached (and the cooling begins), the box remains closed. Therefore the final numbers of kilomoles (n'_b and n'_c) in the box and in the surrounding chamber satisfy the equation

$$\left(\frac{n'_b R}{V_b} - \frac{n'_c R}{V_c} \right) T' = \frac{M_{\ell} g}{A_{\ell}}$$

Since the total number of kilomoles in the system is constant, we also have

$$n'_c = n_b + n_c - n'_b$$

Combining these two displayed equations, we obtain

$$n'_b V_c - (n_b + n_c - n'_b) V_b = \frac{M_{\ell} g V_b V_c}{A_{\ell} R T'}$$

Solving for n'_b , we find

$$n'_b = \frac{(M_{\ell} g V_b V_c / A_{\ell} R T') + (n_b + n_c) V_b}{V_b + V_c}$$

(f) Examining the final expression in part (e), we see that in the limit $T' \rightarrow \infty$, the first term in the numerator approaches zero, so that $n'_b \rightarrow (n_b + n_c) V_b / (V_b + V_c)$, as desired. That is,

$$\frac{n'_b N_A}{V_b} \rightarrow \frac{(n_b + n_c) N_A}{(V_b + V_c)}$$

where N_A is Avogadro's number. The expression on the left is the final number density of gas molecules in the box, while the expression on the right is the overall average number density for the whole system. These approach one another because the density difference corresponding to a given fixed pressure difference of $M_{\ell} g/A_{\ell}$ drops to essentially zero as $T' \rightarrow \infty$.

17-29

No. It would be tempting (but incorrect) to suppose that Bernoulli's equation is applicable to this problem. Bernoulli's equation expresses energy conservation for a moving fluid in the absence of

heat conduction and/or internal energy release or consumption. Very evidently, when air flows around a fire and up a chimney, it is subject to considerable energy input. Bernoulli's equation is inapplicable, and there are not enough other equations to determine the flow speed in the chimney. More directly, on the basis of intuition about the physical situation, it seems evident that the flow speed must depend upon details not included in the information given in the exercise statement. The problem is underdetermined.

17-30

(a) The frequency ν of a simple pendulum is given by

$$\nu = \frac{1}{2\pi} \sqrt{\frac{g}{l}}$$

Differentiating, we find

$$\frac{d\nu}{dl} = \frac{\sqrt{g}}{2\pi} \left(\frac{-1}{2} \right) l^{-3/2} = -\frac{1}{2} \frac{\nu}{l}$$

The fractional frequency decrease is therefore given by

$$\frac{-d\nu}{\nu} = \frac{-d\nu}{dl} \frac{dl}{\nu} = \frac{1}{2} \frac{dl}{l}$$

(b) The increase in length $dl = \alpha l dT$, where α is the coefficient of linear expansion. Therefore $dl/l = \alpha dT$.

(c) Since $dT = 10^\circ\text{C}$ and the coefficient $\alpha = 10.5 \times 10^{-6}/^\circ\text{C}$, the fractional decrease in frequency is $(-d\nu/\nu) = \frac{1}{2}\alpha dT = 0.5(10.5 \times 10^{-6}/^\circ\text{C})(10^\circ\text{C}) = 5.25 \times 10^{-5}$. The number of seconds lost each day is equal to $(-d\nu/\nu)$ times the number of seconds in one day, which is 86,400. Therefore the clock loses $(5.25 \times 10^{-5})(0.864 \times 10^5) = \underline{4.5 \text{ seconds per day}}$.

17-31

(a) Hydrostatic equilibrium of the liquid in the horizontal section of the tube implies that $\rho_t g h_t = \rho_o g h_o$, where ρ_t is the density of the liquid in the warm column and ρ_o is the density of the liquid in the cold column. Using the definition of the coefficient of volume expansion γ , the volumes of a given mass M of liquid at temperatures t and 0°C are related by $V_t = V_o(1 + \gamma t)$. Since $\rho_t V_t = \rho_o V_o$, the densities are related by $\rho_o = \rho_t(1 + \gamma t)$. But then, since $\rho_t h_t = \rho_o h_o$, we have $h_t = h_o(1 + \gamma t)$. Solving this for γ , we find $\gamma = (h_t - h_o)/(h_o t)$, as desired.

(b) With $h_t - h_o = 1.0 \text{ cm}$, $h_o = 100 \text{ cm}$, and $t = 20^\circ\text{C}$, we obtain $\gamma = (1.0)/[(100)(20)] = \underline{500 \times 10^{-6}/^\circ\text{C}}$.

Assuming that the torsion constant k does not vary with temperature, the change in frequency $d\nu$ corresponding to a change in the gyration radius of the balance wheel is given by

$$\frac{d\nu}{\nu} = \frac{1}{\nu} \frac{d\nu}{dG} dG = \frac{1}{\nu} \left(\frac{-1}{2\pi G^2} \sqrt{\frac{k}{M}} \right) dG = \frac{-dG}{G}$$

Since the balance wheel is homogeneous in composition, it changes in size but not in shape in response to temperature variations. Therefore its gyration radius G varies with temperature in the same manner that any of its linear dimensions do:

$$\frac{dG}{G} = \alpha dT$$

Here α is the coefficient of linear expansion. Assuming that αdT is much less than unity, and that α does not vary rapidly with temperature, the above equations are sufficiently accurate. Then $d\nu/\nu = -\alpha dT$ and the number of seconds gained per day is $(86,400 \text{ sec/day}) (d\nu/\nu) = (8.64 \times 10^4 \text{ sec/day}) (-\alpha dT)$. In the present problem, $dT = -50 \text{ }^\circ\text{C}$. If the balance wheel were made of steel, we would have $\alpha = 10.5 \times 10^{-6}/^\circ\text{C}$, and therefore the watch would gain $(8.64 \times 10^4) (-10.5 \times 10^{-6}) (-50) = \underline{45.4 \text{ seconds per day}}$. If the balance wheel were made of Invar, for which $\alpha = 0.9 \times 10^{-6}/^\circ\text{C}$, the watch would gain $(8.64 \times 10^4) (-0.9 \times 10^{-6}) \times (-50) = \underline{3.9 \text{ seconds per day}}$.

Denoting the radius of the capillary by r and its cross-sectional area by A , the increase dr due to thermal expansion is given by $dr = r\alpha dT$, where α is coefficient of linear expansion of the tube material, and dT is the temperature increase. We assume that $dr/r \ll 1$. Since $A = \pi r^2$, we find that $dA/A = 2dr/r$, or that $dA \equiv A\beta dT = A(2\alpha)dT$, where β is called the area coefficient of expansion. If the temperature is changed from T to $T' \equiv T + dT$, the cross-sectional area changes from A to $A' \equiv A + dA = A(1 + 2\alpha dT)$. The change in length (from L to $L' \equiv L + dL$) of the liquid thread is governed by the volume expansion of the liquid. Since $V' = V(1 + \gamma dT)$, where γ is the coefficient of volume expansion for the liquid, we have $L'A' = V' = V(1 + \gamma dT) = LA(1 + \gamma dT)$. But $A'/A = 1 + 2\alpha dT$, so

$$L' = L \frac{(1 + \gamma dT)}{(1 + 2\alpha dT)} = L \left\{ [1 + (\gamma - 2\alpha)dT + O[(\alpha dT)^2]] \right\}$$

For a temperature increase ΔT , the length change $\Delta L \equiv L' - L \simeq L(\gamma - 2\alpha)\Delta T$, and the relative change in length is

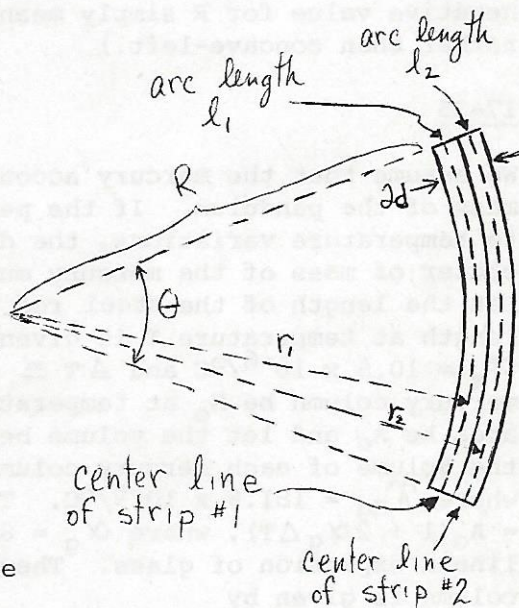
$$\frac{\Delta L}{L} \approx (\gamma - 2\alpha)\Delta T$$

Here we have assumed that $\alpha \Delta T \ll 1$. (NOTE: The specification that a ruler is used for the measurement is to avoid the use of scale that expands with the tube.)

17-34

For definiteness, we let T_0 denote the temperature at which the strips have the same length l_0 (and are therefore straight). For $T > T_0$, the strips have lengths l_1 and l_2 which both exceed l_0 . (The expansion coefficients α_1 and α_2 are both positive.) Unless $\alpha_1 = \alpha_2$, the bimetallic strip will assume a curved shape; the side with the smaller coefficient will be nearer to the center of curvature.

There certainly are stresses along the weld between the strips, but the center line of each strip is free from compressive stress. Therefore the center line of each strip expands freely, and its length is given by $l_i = l_0(1 + \alpha_i \Delta T)$ where $i = 1, 2$, and $\Delta T \equiv T - T_0$. For convenience, we suppose that $\alpha_2 > \alpha_1$ and $T > T_0$; Figure 17E-34 and the figure at right are appropriate for this case. The curved strip extends along an arc of central angle θ . The radii r_1 and r_2 of the center lines of the strips are $r_1 = R + d/2$ and $r_2 = R + 3d/2$. Then the difference in the lengths of those center lines is given by



$$r_2\theta - r_1\theta = \theta d \quad (1)$$

But the difference in length is also given by

$$\begin{aligned} l_2 - l_1 &= l_0(1 + \alpha_2 \Delta T) - l_0(1 + \alpha_1 \Delta T) \\ &= l_0(\alpha_2 - \alpha_1)\Delta T \end{aligned} \quad (2)$$

From eqs. (1) and (2), we find that the angle is given by

$$\theta = \frac{l_0(\alpha_2 - \alpha_1)\Delta T}{d} \quad (3)$$

As can be seen from the figure, the angle θ is also equal to the length l_1 of the center section of strip 1, divided by $r_1 = R + d/2$:

$$\theta = \frac{l_1}{R + d/2} = \frac{l_0(1 + \alpha_1 \Delta T)}{R + d/2} \approx \frac{l_0}{R} \quad (4)$$

where we have assumed that $\alpha_1 \Delta T \ll 1$ and $d \ll R$. Combining eqs. (3) and (4) and solving for R , we obtain

$$R = \frac{d}{(\alpha_2 - \alpha_1) \Delta T} \quad (5)$$

as desired. Although we have assumed that $\alpha_2 > \alpha_1$ and $\Delta T > 0$, it is not difficult to recognize that eq. (5) is valid for all combinations of algebraic signs of $\alpha_2 - \alpha_1$ and ΔT . (NOTE: A negative value for R simply means that the strip is concave-right rather than concave-left.)

17-35

We assume that the mercury accounts for essentially all of the mass of the pendulum. If the pendulum's period is to be insensitive to temperature variations, the distance from the pivot to the center of mass of the mercury must be independent of temperature. Let the length of the steel rod be L_0 at temperature T_0 ; its length at temperature T is given by $L = L_0(1 + \alpha_s \Delta T)$, where $\alpha_s = 10.5 \times 10^{-6}/^\circ\text{C}$ and $\Delta T \equiv T - T_0$. Let the height of each mercury column be H_0 at temperature T_0 ; let the cross-sectional area be A_0 and let the volume be $V_0 \equiv A_0 H_0$. At temperature T , the volume of each mercury column is given by $V = V_0(1 + \gamma_{\text{Hg}} \Delta T)$, where $\gamma_{\text{Hg}} = 181.9 \times 10^{-6}/^\circ\text{C}$. The cross-sectional area $A = A_0(1 + 2\alpha_g \Delta T)$, where $\alpha_g = 8.5 \times 10^{-6}/^\circ\text{C}$, the coefficient of linear expansion of glass. Therefore the height H of the mercury column is given by

$$H = \frac{V}{A} = \frac{H_0(1 + \gamma_{\text{Hg}} \Delta T)}{(1 + 2\alpha_g \Delta T)} \approx H_0 [1 + (\gamma_{\text{Hg}} - 2\alpha_g) \Delta T] \quad (1)$$

where we have assumed $\alpha_g \Delta T \ll 1$. At temperature T_0 , the distance from the pivot to the center of mass is $L_0 - H_0/2$; at temperature T , it is $L - H/2$. If the clock is to be insensitive to temperature, we must have

$$\begin{aligned} L_0(1 + \alpha_s \Delta T) - \frac{H_0}{2} [1 + (\gamma_{\text{Hg}} - 2\alpha_g) \Delta T] \\ = L_0 - H_0/2 \end{aligned} \quad (2)$$

Equation (2) implies that

$$L_o \alpha_s - H_o / 2 (\gamma_{Hg} - 2 \alpha_g) = 0 \quad (3)$$

Solving eq. (3) for H_o , we obtain

$$H_o = \frac{2 \alpha_s L_o}{(\gamma_{Hg} - 2 \alpha_g)} \quad (4)$$

With $L_o = 80.0$ cm, we find

$$H_o = \frac{2(10.5 \times 10^{-6})(80.0)}{(181.9 \times 10^{-6} - 17.0 \times 10^{-6})} = \underline{10.2 \text{ cm}}$$

17-36

We begin by determining the length that each rod would have if it could expand freely. Using subscripts a and s for aluminum and steel, we have

$$l_a = l_o (1 + \alpha_a \Delta t) \quad (1)$$

and

$$l_s = l_o (1 + \alpha_s \Delta t) \quad (2)$$

where $l_o = 50$ cm, $\alpha_a = 25.5 \times 10^{-6}/^\circ\text{C}$, $\alpha_s = 10.5 \times 10^{-6}/^\circ\text{C}$, and $\Delta t = 20$ °C. To remain fitted together as described, each rod must be compressed somewhat; we denote the lengths by l'_a and l'_s . If we let $x \equiv l'_s - l'_a$, then $l'_a = l_o - x$, since we must have $l'_a + l'_s = l_o + l_o = 2l_o$. The compression of the steel rod is

$$l_s - l'_s = l_o \alpha_s \Delta t - x \quad (3)$$

while the aluminum rod is compressed by

$$l_a - l'_a = l_o \alpha_a \Delta t + x \quad (4)$$

Since the rods have the same cross-sectional area, the law of action and reaction implies that the compressive stresses are equal. That is, $\sigma_a = \sigma_s$, or

$$Y_a \frac{(l_a - l'_a)}{l_a} = Y_s \frac{(l_s - l'_s)}{l_s} \quad (5)$$

Using eqs. (1) - (4), we can rearrange eq. (5) to obtain

$$\frac{Y_s l_a}{Y_a l_s} = \frac{Y_s (1 + \alpha_a \Delta t)}{Y_a (1 + \alpha_s \Delta t)} = \frac{l_a - l'_a}{l_s - l'_s}$$

$$= \frac{l_o \alpha_a \Delta t + x}{l_o \alpha_s \Delta t - x} \quad (6)$$

Using $Y_s = 21 \times 10^{10} \text{ N/m}^2$, $Y_a = 7.0 \times 10^{10} \text{ N/m}^2$, and the various other numerical values, we have

$$\left(\frac{21 \times 10^{10}}{7.0 \times 10^{10}} \right) \frac{(1 + 5.1 \times 10^{-4})}{(1 + 2.1 \times 10^{-4})}$$

$$= \frac{25.5 \times 10^{-3} \text{ cm} + x}{10.5 \times 10^{-3} \text{ cm} - x} \quad (7)$$

Solving eq. (7) for x , we find

$$x = \frac{1}{4.0009} [(3.0009)(10.5) - 25.5] \times 10^{-3} \text{ cm}$$

$$= 1.502 \times 10^{-3} \text{ cm}$$

Using this in eq. (3), we have

$$l_s - l'_s = (50)(10.5 \times 10^{-6})(20) - 1.502 \times 10^{-3}$$

$$= 8.998 \times 10^{-3} \text{ cm}$$

Finally we apply eq. (5) to obtain

$$\sigma_a = \sigma_s = \frac{Y_s (l_s - l'_s)}{l_s}$$

$$= \frac{(21 \times 10^{10} \text{ N/m}^2)(8.998 \times 10^{-3} \text{ cm})}{50.0(1 + 2.1 \times 10^{-4})}$$

$$= \underline{3.78 \times 10^7 \text{ N/m}^2} = \underline{373 \text{ atm}}$$

17-37

The heat energy required is given by

$$\begin{aligned}
 \Delta H &= \int_{T_i}^{T_f} C(T) dT \\
 &= m \int_{T_i}^{T_f} (\kappa_1 T + \kappa_3 T^3) dT \\
 &= m \left[\frac{\kappa_1}{2} (T_f^2 - T_i^2) + \frac{\kappa_3}{4} (T_f^4 - T_i^4) \right]
 \end{aligned}$$

Inserting the given numerical values, we find

$$\begin{aligned}
 \Delta H &= 15 \left\{ \left(\frac{2.75 \times 10^{-6}}{2} \right) [(30)^2 - (5)^2] \right. \\
 &\quad \left. + \left(\frac{2.48 \times 10^{-7}}{4} \right) [(30)^4 - (5)^4] \right\} \\
 &= 0.0180 + 0.7527 = \underline{0.771 \text{ cal}} = 7.71 \times 10^{-4} \text{ kcal.}
 \end{aligned}$$

CHAPTER EIGHTEEN

18-6

Letting subscripts 1 and 2 refer to gold and silver, respectively, we read the capacities per atom from Table 18-3: $c_1 = 2.99\text{k}$ and $c_2 = 3.06\text{k}$. (The value for gold is correct at 300 K while that for silver refers to 400 K; we neglect any possible variations of the heat capacities over the temperature range of interest here.) The heat required to raise a mass m of an element by ΔT is given by $\Delta H = c' (mN_A/W) \Delta T$, where $N_A = 6.02 \times 10^{26}$ atoms/kmol and W is the atomic "weight" of the element. We use a general chemistry or physics reference book to find $W_1 = 197.0$ kg/kmol and $W_2 = 107.9$ kg/kmol. With $m = 1 \text{ g} = 10^{-3} \text{ kg}$ and $\Delta T = 1 \text{ }^\circ\text{C} = 1 \text{ K}$, we have

$$\begin{aligned}
 \Delta H_1 &= \frac{(2.99\text{k/atom})(10^{-3}\text{kg})(6.02 \times 10^{26}\text{atoms/kmol})(1 \text{ K})}{(197.0 \text{ kg/kmol})} \\
 &= (9.137 \times 10^{21})(1.38 \times 10^{-23})\text{J} = \underline{0.126 \text{ J}}
 \end{aligned}$$

and

$$\begin{aligned}
 \Delta H_2 &= \frac{(3.06\text{k/atom})(10^{-3}\text{kg})(6.02 \times 10^{26}\text{atoms/kmol})(1 \text{ K})}{(107.9 \text{ kg/kmol})} \\
 &= (1.707 \times 10^{22})(1.39 \times 10^{-23})\text{J} = \underline{0.236 \text{ J}}
 \end{aligned}$$