22-17

When the two cells of Fig. 22E-3 are reconnected in parallel, the voltages across them are equal. The current in each cell is then inversely proportional to its internal resistance. Using the subscripts Cu for copper and Ag for silver, we have

$$\frac{I_{Cu}}{I_{Ag}} = \frac{R_{Ag}}{R_{Cu}} \tag{1}$$

Using the notation of Section 22-1, the currents are related to the charges transferred and kilomoles deposited in a time interval t by

$$I_{Cu} = \frac{q_{Cu}}{t} = \frac{\nu_{Cu}\, n_{Cu}\, \mathcal{F}}{t} \tag{2}$$

and

$$I_{Ag} = \frac{q_{Ag}}{t} = \frac{\nu_{Ag}\, n_{Ag}\, \mathcal{F}}{t} \tag{3}$$

The masses deposited are given by $\Delta m_{Cu} = \nu_{Cu} M_{Cu}$ and $\Delta m_{Ag} = \nu_{Ag} M_{Ag}$, where M_{Cu} and M_{Ag} are the atomic weights of copper and silver. From eqs. (2) and (3), we find the ratio of currents to be given by

$$\frac{I_{Cu}}{I_{Ag}} = \frac{\nu_{Cu}\, n_{Cu}}{\nu_{Ag}\, n_{Ag}} = \frac{\nu_{Cu}}{\nu_{Ag}} \frac{(\Delta m_{Cu}/M_{Cu})}{(\Delta m_{Ag}/M_{Ag})} \tag{4}$$

We have $\nu_{Cu} = 2$, $\nu_{Ag} = 1$, $M_{Cu} = 63.54$ kg/kmol and $M_{Ag} = 107.88$ kg/kmol. In order to obtain equal mass depositions, eqs. (1) and (4) require that

$$\frac{R_{Ag}}{R_{Cu}} = \frac{I_{Cu}}{I_{Ag}} = \left(\frac{\nu_{Cu}}{\nu_{Ag}}\right)\left(\frac{M_{Ag}}{M_{Cu}}\right)$$

$$= \left(\frac{2}{1}\right)\left(\frac{107.88}{63.54}\right) = 3.40$$

The internal resistance of the silver cell should be 3.40 times that of the copper cell.

22-18

The number density of valence electrons in solid silver is

$$n = \frac{\nu A \rho_m}{M} = \frac{(1)(6.02 \times 10^{26}/\text{kmol})(1.05 \times 10^4 \text{ kg/m}^3)}{108 \text{ kg/kmol}}$$

$$= 5.85 \times 10^{28} \text{ m}^{-3}$$

Therefore the charge density due to the valence electrons is

$$|\rho| = ne = (5.85 \times 10^{28} \text{ m}^{-3})(1.602 \times 10^{-19} \text{ C})$$

$$= 9.37 \times 10^9 \text{ C/m}^3 \qquad (1)$$

The electron drift speed is given by

$$v_d = \frac{i}{|\rho| a} \qquad (2)$$

With $i = 2.4$ A, $a = 3.0 \times 10^{-6}$ m^2, and $|\rho|$ as given in eq. (1), eq. (2) yields

$$v_d = \frac{2.4}{(9.37 \times 10^9)(3.0 \times 10^{-6})} = \underline{8.54 \times 10^{-5} \text{ m/s}}$$

22-19

We denote the initial and final diameters and lengths by d_1, d_2, ℓ_1, and ℓ_2, respectively. Since there is no change in volume, we have $\pi d_1^2 \ell_1/4 = \pi d_2^2 \ell_2/4$. With $d_1 = 1.00$ cm, $d_2 = 0.100$ cm, and $\ell_1 = 1.00$ m, we find that the final length is given by

$$\ell_2 = \ell_1 \frac{d_1^2}{d_2^2} = 1.00 \text{ m} \left(\frac{1.00}{0.100}\right)^2 = \underline{100 \text{ m}}$$

In terms of the conductivity σ of the metal, the rod's resistance $R_1 = \ell_1/(\pi d_1^2 \sigma/4)$. Since σ is assumed to remain constant during the stretching, the resistance ratio is

$$\frac{R_2}{R_1} = \frac{(4\ell_2/\pi d_2^2 \sigma)}{(4\ell_1/\pi d_1^2 \sigma)}$$

$$= \frac{\ell_2/\ell_1}{(d_2/d_1)^2} = \frac{(100/1.00)}{(0.100/1.00)^2}$$

$$= \underline{1.00 \times 10^4}$$

22-20

Since the wires are in series, they carry the same current:

$i_1 = i_2$. The current densities differ because of the unequal cross-sectional areas:

$$\frac{j_2}{j_1} = \frac{(i_2/a_2)}{(i_1/a_1)} = \frac{a_1}{a_2} = \underline{2}$$

The resistances of the wires differ, even though both wires have length ℓ and resistivity ρ:

$$\frac{R_2}{R_1} = \frac{(\rho \ell /a_2)}{(\rho \ell /a_1)} = \frac{a_1}{a_2} = 2$$

Therefore the potential differences across the two wires also differ:

$$\frac{V_2}{V_1} = \frac{i_2 R_2}{i_1 R_1} = \frac{R_2}{R_1} = \underline{2}$$

Finally, the electric fields are unequal:

$$\frac{E_2}{E_1} = \frac{(V_2/\ell)}{(V_1/\ell)} = \frac{V_2}{V_1} = \underline{2}$$

22-21

We suppose that the resistive force on a charge carrier has the form $\vec{F}_D = -b\vec{v}$, where b is a positive constant and $\vec{v}$ is the velocity of the charge carrier. In the presence of an applied electric field $\vec{\mathcal{E}}$, each carrier will attain a terminal velocity $\vec{v}_t$ such that $\vec{F}_D + q\vec{\mathcal{E}} = -b\vec{v}_t + q\vec{\mathcal{E}} = \vec{0}$. That is, $\vec{v}_t = q\vec{\mathcal{E}}/b$. Letting N denote the number density of charge carriers, the steady-state current density is

$$\vec{j} = Nq\vec{v}_t = \frac{Nq^2\vec{\mathcal{E}}}{b}$$

This equation agrees with Eq. (22-31) provided that we set $\sigma = Nq^2/b$, or $b = Nq^2/\sigma$.

22-22

(a) Applying Ohm's law within each wire, we have $\vec{\mathcal{E}}_1 = \vec{j}/\sigma_1 = \hat{x}(j/\sigma_1)$ and $\vec{\mathcal{E}}_2 = \vec{j}/\sigma_2 = \hat{x}(j/\sigma_2)$.

(b) We apply Gauss' law to a right cylinder with end faces parallel to and bracketing the interface at x = 0. We find that the surface charge density Σ on the interface must be

$$\Sigma = \epsilon_o(\vec{\mathcal{E}}_2 \cdot \hat{x} - \vec{\mathcal{E}}_1 \cdot \hat{x})$$

$$= \epsilon_o\left(\frac{j}{\sigma_2} - \frac{j}{\sigma_1}\right) = \epsilon_o j\left(\frac{1}{\sigma_2} - \frac{1}{\sigma_1}\right)$$

$$= \frac{\epsilon_o j(\sigma_1 - \sigma_2)}{\sigma_1 \sigma_2} \qquad (1)$$

(c) If the charge layer has a finite thickness δ, the average charge density within the layer is

$$\bar{\rho} \equiv \frac{1}{\delta}\int \rho(x)\,dx = \frac{\Sigma}{\delta} = \frac{\epsilon_o j(\sigma_1 - \sigma_2)}{\delta \sigma_1 \sigma_2} \qquad (2)$$

(d) With $\delta = 1.0 \times 10^{-9}$ m, $j = 1.0 \times 10^6$ A/m^2, $\sigma_1 = 1.8 \times 10^7$ S/m and $\sigma_2 = 5.65 \times 10^7$ S/m, we find that

$$\bar{\rho} = \frac{(8.85 \times 10^{-12})(1.0 \times 10^6)[(1.8 - 5.65) \times 10^7]}{(1.0 \times 10^{-9})(1.8 \times 10^7)(5.65 \times 10^7)}$$

$$= \underline{-3.35 \times 10^{-4} \text{ C/m}^3} \qquad (3)$$

(e) The average charge density $\bar{\rho} = -\overline{N'_e}e$, where $\overline{N'_e}$ is the average excess number density of electrons. Using eq. (3), we obtain

$$\overline{N'_e} = \frac{-\bar{\rho}}{e} = \frac{3.35 \times 10^{-4} \text{ C/m}^3}{1.60 \times 10^{-19} \text{ C}}$$

$$= \underline{2.09 \times 10^{15} \text{ m}^{-3}}$$

The number densities of atoms in the two wires are given by $N_1 = \rho_{m1}A/M_1$ and $N_2 = \rho_{m2}A/M_2$. Here ρ_{m1} and ρ_{m2} are the mass densities, M_1 and M_2 are the atomic weights, and A is Avogadro's number. For the tungsten wire, we have $\rho_{m1} = 1.93 \times 10^4$ kg/m^3 and $M_1 = 184$ kg/kmol, so that

$$N(W) \equiv N_1 = \frac{(1.93 \times 10^4)(6.02 \times 10^{26})}{184} = 6.31 \times 10^{28} \text{ m}^{-3}$$

For the copper wire, we have $\rho_{m2} = 8.94 \times 10^3$ kg/m^3 and $M_2 = 63.5$, so that

$$N(Cu) \equiv N_2 = \frac{(8.94 \times 10^3)(6.02 \times 10^{26})}{63.5} = 8.48 \times 10^{28} \text{ m}^{-3}$$

Therefore, the ratios of average excess electron number density to number density of atoms are $N'_e/N(W) = \underline{3.31 \times 10^{-14}}$ and $N'_e/N(Cu) = \underline{2.47 \times 10^{-14}}$.

22-23

According to Eq. (22-43a), the conductivity is given by $\sigma = Ne^2 \tau/m_e$, where N is the number density of conduction electrons and τ is the mean scattering time. Therefore the conductivity ratio of two substances is $\sigma_1/\sigma_2 = (N_1 \tau_1/N_2 \tau_2)$. If the mean scattering times are assumed to be equal, then $\sigma_1/\sigma_2 = N_1/N_2$. The number density of molecules is given by $N_M = \rho_m A/M$, where is the mass density, M is the molecular weight, and A is Avogadro's number. In copper (an excellent conductor), the number of conduction electrons for each molecule (atom) is about one: $N_{Cu}/(N_M)_{Cu} = 1$. The atomic number density in copper is

$$(N_M)_{Cu} = \frac{(8.9\ g/cm^3)(6.02 \times 10^{23}\ mole^{-1})}{(63.5\ g/mole)}$$

$$= 8.44 \times 10^{22}\ atoms/cm^3 \qquad (1)$$

Using copper as the standard, the relative number density of conduction electrons in each of the other materials is given by

$$\frac{N_2}{N_{Cu}} = \frac{\sigma_2}{\sigma_{Cu}} \qquad (2)$$

The number of conduction electrons per atom or molecule is given by

$$\frac{N_2}{(N_M)_2} = \frac{\sigma_2}{\sigma_{Cu}} N_{Cu} \left(\frac{M_2}{\rho_{m2} A}\right)$$

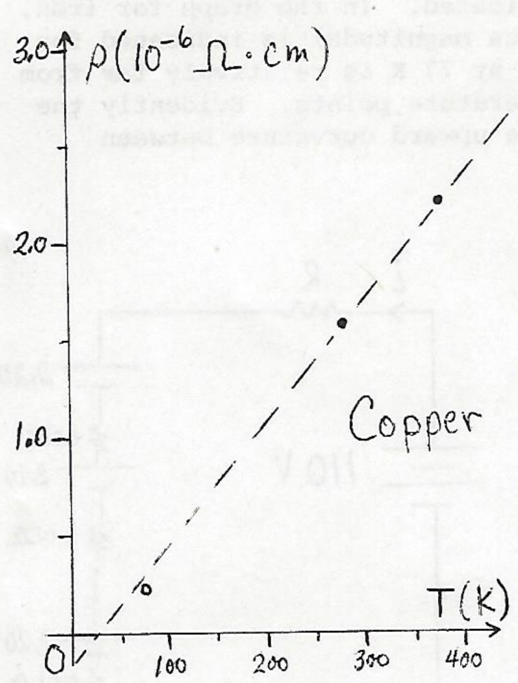
$$= \frac{\sigma_2}{\sigma_{Cu}} (N_M)_{Cu} \left(\frac{M_2}{\rho_{m2} A}\right)$$

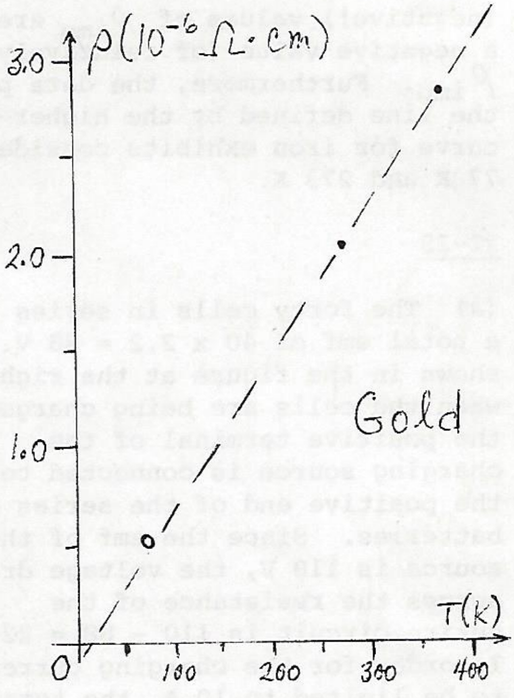
$$= \left(\frac{\sigma_2}{\sigma_{Cu}}\right) \left[\frac{(\rho_m)_{Cu}}{\rho_{m2}}\right] \left(\frac{M_2}{M_{Cu}}\right) \qquad (3)$$

With the numerical values given in the exercise statement, eqs. (2) and (3) yield the values tabulated below.

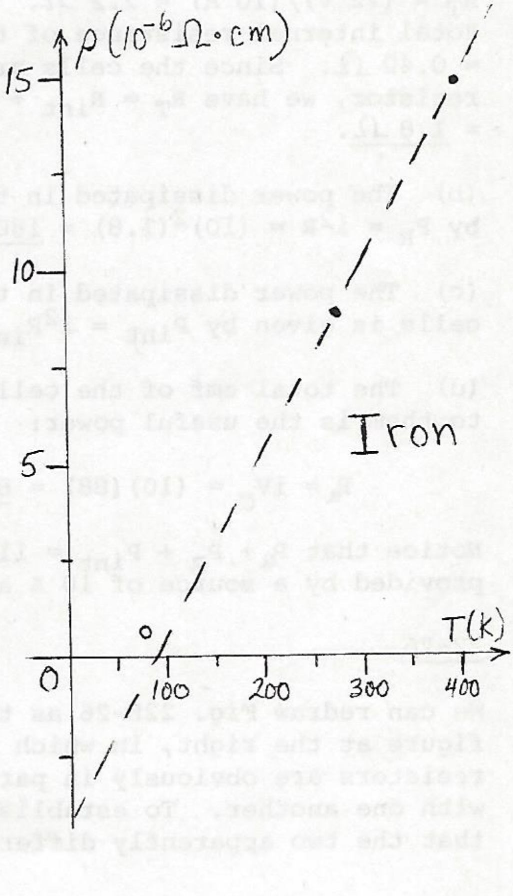
Substance	N/N_{Cu}	N/N_M
copper	1	1
iron	0.17	0.17
silicon	1.7×10^{-10}	2.8×10^{-10}
glass	8.5×10^{-20}	3.1×10^{-19}

22-24



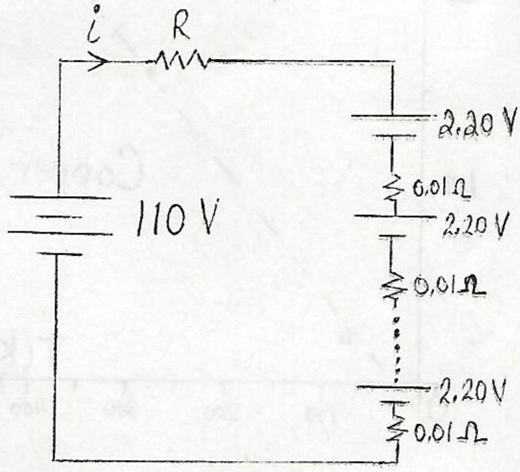


The resistivity data are plotted in the graphs above and at right. In each graph, we have drawn a straight line based on the two higher-temperature points. The slope of the line gives an estimate for the constant A in the expression $\rho = \rho_{imp} + AT$. The numerical value for copper is $A(Cu) = 0.01(2.2 - 1.56) \times 10^{-6} = 6.4 \times 10^{-9}\ \Omega\cdot cm/K = 6.4 \times 10^{-11}\ \Omega\cdot m/K$. The numerical value for gold is $A(Au) = 0.01(2.84 - 2.04) \times 10^{-6} = 8.0 \times 10^{-9}\ \Omega\cdot cm/K = 8.0 \times 10^{-11}\ \Omega\cdot m/K$. The numerical value for iron is $A(Fe) = 0.01(14.7 - 8.9) \times 10^{-6} = 5.8 \times 10^{-8}\ \Omega\cdot cm/K = 5.8 \times 10^{-10}\ \Omega\cdot m/K$. In the graphs for copper and gold, the points at 77 K lie relatively close to the lines defined by the higher-temperature points, so the linear relation would appear to serve reasonably well



in the range 77-373 K. Notice, however, that unrealistic (negative!) values of ρ_{imp} are indicated. In the graph for iron, a negative value (of relatively large magnitude) is indicated for ρ_{imp}. Furthermore, the data point at 77 K is relatively far from the line defined by the higher-temperature points. Evidently the curve for iron exhibits considerable upward curvature between 77 K and 273 K.

22-25



(a) The forty cells in series have a total emf of 40 x 2.2 = 88 V. As shown in the figure at the right, when the cells are being charged, the positive terminal of the charging source is connected to the positive end of the series of batteries. Since the emf of the source is 110 V, the voltage drop across the resistance of the entire circuit is 110 - 88 = 22 V. In order for the charging current to be limited to 10 A, the total resistance of the circuit must be R_T = (22 V)/(10 A) = 2.2 Ω. The total internal resistance of the cells is R_{int} = (40)(0.0100) = 0.40 Ω. Since the cells are in series with the current-limiting resistor, we have $R_T = R_{int} + R$, so that $R = R_T - R_{int}$ = 2.2 - 0.40 = 1.8 Ω.

(b) The power dissipated in the current-limiting resistor is given by $P_R = i^2R = (10)^2(1.8)$ = 180 W.

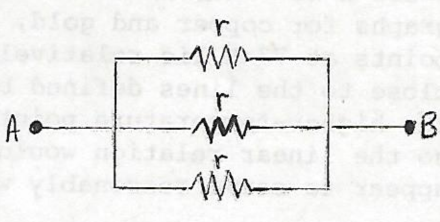
(c) The power dissipated in the internal resistances of the forty cells is given by $P_{int} = i^2R_{int} = (10)^2(0.4)$ = 40 W.

(d) The total emf of the cells is V_C = 88 V. The power delivered to them is the useful power:

$$P_u = iV_C = (10)(88) = 880\ W$$

Notice that $P_u + P_R + P_{int}$ = 1100 W, which is the total power provided by a source of 10 A at 110 V.

22-26

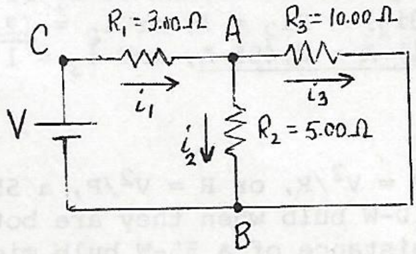


We can redraw Fig. 22E-26 as the figure at the right, in which the resistors are obviously in parallel with one another. To establish that the two apparently different circuits are equivalent, we simply observe that one end of each resistor is connected to terminal A and the other end to terminal B. Hence Eq. (22-54a) implies that the resistance from A to B is

$$\frac{1}{R} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} = \frac{3}{r}$$

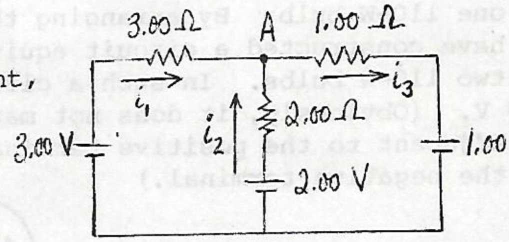
so that $R = \underline{r/3}$.

22-27



The circuit has been redrawn at the right, with labels A, B, and C for the three nodes in the circuit. If $i_3 = 1.00$ A, then the voltage drop between A and B is given by $V_{AB} = i_3R_3 = (1.00\text{ A}) \times (10.00\ \Omega) = 10.0$ V. But $V_{AB} = i_2R_2$, so $i_2 = V_{AB}/R_2 = (10.0\text{ V})/(5.00\ \Omega) = \underline{2.00\text{ A}}$. Therefore $i_1 = i_2 + i_3 = 2.00\text{ A} + 1.00\text{ A} = \underline{3.00\text{ A}}$. Finally, applying the loop rule to CABC, we find that $V - i_1R_1 - i_2R_2 = 0$, so that $V = i_1R_1 + i_2R_2 = (3.00\text{ A})(3.00\ \Omega) + (2.00\text{ A})(5.00\ \Omega) = \underline{19.00\text{ V}}$.

22-28



The circuit is redrawn at the right with arrows indicating the sign convention used for the currents. We apply Kirchoff's loop rule (voltage law) to the left-hand loop of the circuit:

$$3.00\text{ V} - (3.00\ \Omega)i_1 + (2.00\ \Omega)i_2 - 2.00\text{ V} = 0$$

or

$$3i_1 - 2i_2 = 1 \qquad (1)$$

where the currents are measured in amperes. Applied to the right-hand loop, Kirchoff's voltage law yields:

$$2.00\text{ V} - (2.00\ \Omega)i_2 - (1.00\ \Omega)i_3 - 1.00\text{ V} = 0$$

or

$$2i_2 + i_3 = 1 \qquad (2)$$

Applying Kirchoff's node rule (current law) to the node at A yields

$$i_3 = i_1 + i_2 \qquad (3)$$

Using eq. (3), we can eliminate i_3 from eq. (2):

$$2i_2 + (i_1 + i_2) = 1$$

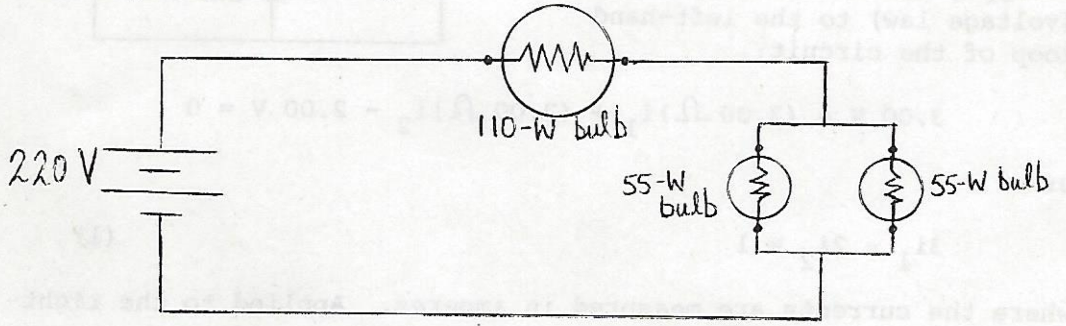
or

$$i_1 + 3i_2 = 1 \qquad (4)$$

Using eq. (4) to eliminate i_1 from eq. (1), we obtain $3(1 - 3i_2) - 2i_2 = 1$, or $i_2 = \underline{(2/11)\ A = 0.182\ A}$. Then $i_1 = 1 - 3i_2 = \underline{(5/11)\ A = 0.455\ A}$, and $i_3 = 1 - 2i_2 = \underline{(7/11)\ A = 0.636\ A}$.

22-29

Since $P = V^2/R$, or $R = V^2/P$, a 55-W bulb has twice the resistance of a 110-W bulb when they are both operating at 110 V. (Since the resistance of a 55-W bulb might well exhibit a temperature dependence different from that of a 110-W bulb, we cannot make definite comparisons of the resistances at other operating voltages.) At any operating voltage, two identical 55-W bulbs have an effective resistance equal to one-half that of a single 55-W bulb at the same voltage. If the operating voltage is 110 V, the effective resistance of the two 55-W bulbs in parallel equals the resistance of one 110-W bulb. By arranging the bulbs as shown in the figure, we have constructed a circuit equivalent to a series combination of two 110-W bulbs. In such a circuit, each bulb operates at 110 V. (Obviously, it does not matter whether the single bulb is adjacent to the positive terminal of the source -- as shown -- or to the negative terminal.)



22-30

(a) We refer to Fig. 22E-30. With S open, the current in the meter is i_1. Since the meter reading does not change when S is closed, the current in the meter is still i_1, which means that no current is diverted from A to B (or vice versa) that is, the current in AB is zero.

(b) Since there is zero current along the conducting path ASB, the voltage drop must be zero: $\underline{V_{AB} = 0}$.

(c) Applying Kirchoff's voltage law to the left-hand loop, we have $0 = i_1R_1 + V_{AB} - i_2R_3 = i_1R_1 - i_2R_3$, so that

$$i_1R_1 = i_2R_3 \qquad (1)$$

Applying the voltage law to the right-hand loop, we find $0 = i_1R_M - i_2R_4 + V_{AB} = i_1R_M - i_2R_4$, so that

$$i_1R_M = i_2R_4 \qquad (2)$$

Dividing eq. (2) by eq. (1), we obtain

$$\frac{R_M}{R_1} = \frac{R_4}{R_3} \qquad (3)$$

so that $\underline{R_M = R_1R_4/R_3}$.

22-31

(a) Applying Kirchoff's voltage law to loop DHBXD, we obtain

$$i(\lambda \ell_X) - V_X = 0 \qquad (1)$$

since the resistance of the portion DH of the uniform wire is $\lambda \ell_X$.

(b) Similarly, applying Kirchoff's voltage law to loop DHCSD, we obtain

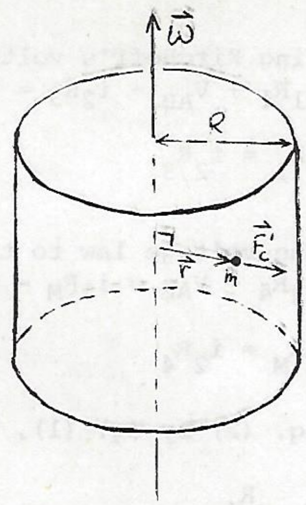
$$i(\lambda \ell_S) - V_S = 0 \qquad (2)$$

(c) Since V_S is known, we can use eqs. (1) and (2) to obtain a proportion in which V_X is the only unknown quantity:

$$\frac{V_X}{V_S} = \frac{i\lambda \ell_X}{i\lambda \ell_S} = \frac{\ell_X}{\ell_S} \qquad (3)$$

(d) The voltages measured are emf's rather than terminal voltages because the currents through the unknown and standard batteries are zero. There is no ohmic voltage drop across the internal resistance in either battery.

22-32



In a noninertial reference frame rotating with the cylinder, there is a centrifugal force acting on any particle not on the axis of cylinder. For a particle of mass m located at displacement $\vec{r}$ from the axis of the cylinder (as shown in the figure at the right), the centrifugal force is given by

$$\vec{F}'_c(r) = \frac{mv^2}{r}\hat{r} = m\omega^2 r\hat{r} \tag{1}$$

Any particle that moves with the cylinder must be subject to a net force of zero in the rotating frame, so a real (not fictitious) centripetal force $-\vec{F}'_c = -m\omega^2 r\hat{r}$ must also act on every particle of the cylinder. For the atomic nuclei and the tightly bound electrons in the material of the cylinder, the centripetal force is provided by neighboring atoms. (For a solid cylinder of normal rigidity rotating at a reasonable rate, such forces are provided at the expense of minuscule deviations from the normal size and shape of the cylinder.) However, the conduction electrons are mobile, and unless some other force cancels $\vec{F}'_c$, those electrons would collect at the periphery of the cylinder ($r = R$). There is a very slight outward shift of the conduction electrons when the cylinder is started spinning. This outward drift ceases when there is an outward electric field $\vec{\mathcal{E}}(r) = \mathcal{E}(r)\hat{r}$ such that at each location a conduction electron experiences zero net force in the rotating frame. Using eq. (1), this happens when

$$0 = -e\vec{\mathcal{E}}(r) + \vec{F}'_c(r) = -e\,\mathcal{E}(r)\hat{r} + m\omega^2 r\hat{r} \tag{2}$$

Therefore $\mathcal{E}(r) = m\omega^2 r/e$, which implies that there is a potential difference between the axis and radius r given by

$$V(r) - V(0) = -\int_0^r \vec{\mathcal{E}}(r')\cdot d\vec{r}\,' = \frac{-m\omega^2}{e}\int_0^r r'dr'$$

$$= -\frac{1}{2}\left(\frac{m}{e}\right)\omega^2 r^2 \tag{3}$$

Therefore the magnitude of the potential difference between the axis and the periphery is

$$\left|V(R) - V(0)\right| = \frac{1}{2}\left(\frac{m}{e}\right)\cdot\omega^2 R^2 \tag{4}$$

as was to be shown. Equation (3) implies that the axis is at a positive potential with respect to the periphery. The prescriptions for transforming the electric field and the electric potential from one reference frame to another (when the frames are in relative motion) have not been presented at this stage in the text. Here we simply point out that if the relative speeds are much less than the speed of light, we can adopt expressions (2) - (4) as also valid in the "laboratory frame" (in which the cylinder is spinning with angular speed ω). [NOTE: A completely acceptable alternative solution can be constructed working entirely in the laboratory frame. As long as $\omega R \ll c$ (so that magnetic effects are negligible), the results are in complete agreement with eqs. (2) - (4) above.]

22-33

(a) In the water, the current density $\vec{j} = \sigma \vec{\mathcal{E}}_{int}$, where $\vec{\mathcal{E}}_{int}$ is the true interior electric field (see Sec. 21-8). Referring to Fig. 22E-33, the total current from the inner conductor to the outer one is given by

$$i = \int_S \vec{j} \cdot d\vec{a} = \sigma \int_S \vec{\mathcal{E}}_{int} \cdot d\vec{a} \qquad (1)$$

where S is a surface within the water that surrounds the inner sphere. Since the water has dielectric constant $K_e \gg 1$, a much larger free charge q must be present on the inner sphere (in order to maintain the given potential difference V of the source) than there would be if the filler had a dielectric constant of unity. In this situation, Gauss' law takes the form

$$q = K_e \epsilon_o \int_S \vec{\mathcal{E}}_{int} \cdot d\vec{a} \qquad (2)$$

Since $q = C'V$, where C' is the capacitance of the system with the water present, eq. (2) yields

$$V = \frac{K_e \epsilon_o}{C'} \int_S \vec{\mathcal{E}}_{int} \cdot d\vec{a} \qquad (3)$$

Equations (1) and (3) imply that the resistance of the system is

$$R \equiv \frac{V}{i} = \frac{K_e \epsilon_o}{\sigma C'} \qquad (4)$$

However, as described in the final paragraph of Chapter 21, the capacitance $C' = K_e C$, where C is the capacitance of the system in the absence of the water. Therefore eq. (3) becomes

$$V = \frac{\epsilon_o}{C} \int_S \vec{\mathcal{E}}_{int} \cdot d\vec{a} \qquad (5)$$

and we can rewrite eq. (4) as

$$R = \frac{\epsilon_o}{\sigma C} \qquad (6)$$

which was to be shown. [NOTE: A perfectly acceptable alternative solution can be based on the recognition that (for a given potential difference), the electric field has exactly the same form as it would have with a vacuum between the spheres. One advantage of the solution given above is that we can see explicitly from eq. (4) that the resistance equals the inverse of the conductivity, times the ratio of the permittivity ($K_e \epsilon_o$) to the capacitance C', no matter what value the dielectric constant K_e might have. Hopefully, it helps to make clear that R depends only on the conductivity of the filler and the geometrical arrangement of the "terminals", and not at all on the dielectric constant of the filler.]

(b) Using eq. (6), we obtain

$$R = \frac{\epsilon_o}{\sigma C} = \frac{\epsilon_o (r_2 - r_1)}{\sigma 4\pi \epsilon_o r_2 r_1} = \frac{1}{4\pi\sigma} \frac{(r_2 - r_1)}{r_2 r_1} \qquad (7)$$

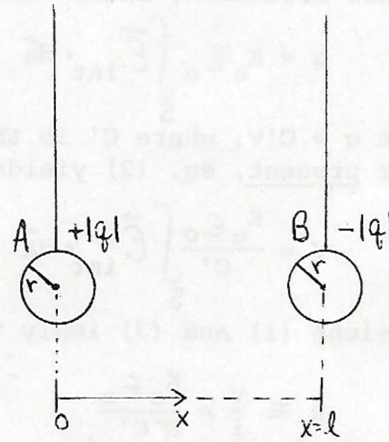
With the given numerical values, the resistance is

$$R = \frac{(0.10 - 0.05)\ \mathrm{m}}{4\pi(1.0 \times 10^{-3}\ \mathrm{S/m})(0.10\ \mathrm{m})(0.050\ \mathrm{m})}$$

$$= \underline{796\ \Omega}$$

22-34

As a matter of convenience and also because the exercise statement refers to the current flowing from A to B, we suppose that the positive terminal of the battery is connected to sphere A. The radius r of each sphere is much smaller than the separation ℓ between their centers, as shown in the figure at the right.



(a) Using the notation indicated in the figure, we calculate the electric field along the line of centers. We are setting out to determine the vacuum capacitance C, so we can neglect the earth surrounding the spheres. Because $\ell \gg r$, the charge distribution on each sphere is not significantly affected by the charge on the other sphere. Therefore each sphere's charge is uniformly distributed over its surface. Then the field between the spheres is

$$\vec{\mathcal{E}}(x) = \frac{\hat{x}}{4\pi\epsilon_0}\left[\frac{|q|}{x^2} + \frac{|q|}{(\ell - x)^2}\right] \tag{1}$$

for $r < x < \ell - r$

Therefore the potential difference is

$$V_A - V_B = -\int_B^A \vec{\mathcal{E}}\cdot d\vec{s} = -\frac{|q|}{4\pi\epsilon_0}\int_{\ell-r}^{r}\left[\frac{1}{x^2} + \frac{1}{(\ell - x)^2}\right]dx$$

$$= \frac{|q|}{4\pi\epsilon_0}\left[\frac{1}{x} - \frac{1}{(\ell - x)}\right]_{x=\ell-r}^{x=r}$$

$$= \frac{|q|}{4\pi\epsilon_0}\left\{\left[\frac{1}{r} - \frac{1}{\ell - r}\right] - \left[\frac{1}{\ell - r} - \frac{1}{\ell - (\ell - r)}\right]\right\}$$

$$= \frac{2|q|}{4\pi\epsilon_0}\left(\frac{1}{r} - \frac{1}{\ell - r}\right) \tag{2}$$

Since $\ell >> r$, eq. (2) can be simplified:

$$V_A - V_B \simeq \frac{|q|}{2\pi\epsilon_0 r} \tag{3}$$

(b) By definition, $C = |q|/|V_A - V_B|$, so eq. (3) implies that the vacuum capacitance of the system is $C = 2\pi\epsilon_0 r$.

(c) Supposing that the result of Exercise 22-33 can be generalized, we find

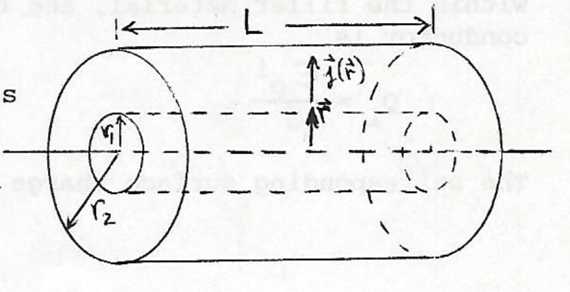
$$R = \frac{\epsilon_0}{\sigma C} = \frac{1}{2\pi\sigma r} \tag{4}$$

[NOTE: Strictly speaking, eq. (4) gives the resistance only when the spheres are immersed in uniform conducting material that fills all space. However, eq. (4) relates σ and the measured resistance R with reasonable accuracy provided that the spheres are buried at a depth comparable to or greater than ℓ (and large compared to r).]

(d) From eq. (4), we have $\sigma = 1/(2\pi R r)$.

22-35

The figure at the right illustrates the situation.



(a) Symmetry considerations imply that the current density $\vec{j}$ is radial and has a magnitude depending only upon r: $\vec{j}(r) = j(r)\hat{r}$. The flux of $\vec{j}$ through a cylinder of radius r coaxial with the resistor must equal the total current:

$$i = \int \vec{j} \cdot d\vec{a} = j(r) \cdot 2\pi rL$$

for $r_1 < r < r_2$. Therefore we have

$$\vec{j}(\vec{r}) = \frac{i}{2\pi rL}\hat{r} \quad (1)$$

for $r_1 < r < r_2$.

(b) Since the material has conductivity σ, the electric field for $r_1 < r < r_2$ is given by

$$\vec{\mathcal{E}}(\vec{r}) = \mathcal{E}(r)\hat{r} = \frac{\vec{j}(\vec{r})}{\sigma} = \frac{i}{2\pi r\sigma L}\hat{r} \quad (2)$$

(c) The potential difference is given by

$$V(r_1) - V(r_2) = -\int_{r_2}^{r_1} \vec{\mathcal{E}}(r) \cdot d\vec{r}$$

$$= \int_{r_1}^{r_2} \vec{\mathcal{E}}(r) \cdot d\vec{r}$$

$$= \frac{i}{2\pi\sigma L}\int_{r_1}^{r_2} \frac{dr}{r} = \frac{i}{2\pi\sigma L}\ln\left(\frac{r_2}{r_1}\right) \quad (3)$$

Therefore the resistance is

$$R \equiv \frac{V}{i} = \frac{1}{2\pi\sigma L}\ln\left(\frac{r_2}{r_1}\right) \quad (4)$$

(d) With $\mathcal{E}(r) = 0$ for $r > r_2$, Gauss' law implies that the cylindrical resistor has zero overall charge. For a coaxial cylindrical surface of radius $r > r_1$, we find

$$\epsilon_0 \int \vec{\mathcal{E}} \cdot d\vec{a} = \epsilon_0 \left(\frac{i}{2\pi r\sigma L}\right)(2\pi rL) = \frac{\epsilon_0 i}{\sigma} \quad (5)$$

Since this is independent of r for $r > r_1$, we can conclude (with the help of Gauss' law) that the volume charge density is zero within the filler material, and that the total charge on the inner conductor is

$$Q_1 = \frac{\epsilon_0 i}{\sigma} \quad (6)$$

The corresponding surface charge density is

$$\Sigma_1 = \frac{Q_1}{2\pi r_1 L} = \frac{\epsilon_o i}{2\pi r_1 \sigma L} \qquad (7)$$

Since the overall charge is zero and the filler has zero charge density, the total charge on the outer conductor is

$$Q_2 = -Q_1 = -\frac{\epsilon_o i}{\sigma} \qquad (8)$$

The corresponding surface charge density is

$$\Sigma_2 = -\frac{\epsilon_o i}{2\pi r_2 \sigma L} \qquad (9)$$

22-36

(a) The vapor is contained in a box 10 cm on a side, so the cross-sectional area of the current-carrying region is $a = 10^{-2}\ m^2$. Assuming that the current density is uniform over the cross section, the current is $i = ja = \sigma\mathcal{E}a$. But the electric field $\mathcal{E} = \Delta V/h$, where $\Delta V = 50$ V and $h = 0.1$ m is the height of the chamber. Therefore $i = \sigma(\Delta V)a/h$, or

$$\sigma = \frac{ih}{(\Delta V)a} = \frac{(1.0 \times 10^{-6}\ A)(0.1\ m)}{(50\ V)(0.01\ m^2)}$$

$$= \underline{2.0 \times 10^{-7}\ S/m}$$

(b) The mean scattering time is given by $\tau = \lambda/v_{rms}$, where λ is the mean free path. The rms thermal speed is given by

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

where m is the mass of the molecule. The molecular weight of methyl alcohol (CH_3OH) is 32, so $m = (32\ kg/kmol) \div (6.02 \times 10^{26}\ kmol^{-1}) = 5.32 \times 10^{-26}$ kg. Thus, at a temperature of 300 K the thermal speed is

$$v_{rms} = \sqrt{\frac{3(1.38 \times 10^{-23})(300)}{(5.32 \times 10^{-26})}}$$

$$= 4.83 \times 10^2\ m/s$$

Given a mean free path $\lambda = 1.0 \times 10^{-5}$ cm $= 1.0 \times 10^{-7}$ m, the mean scattering time is

$$\tau = \frac{\lambda}{v_{rms}} = \frac{1.0 \times 10^{-7}\ m}{4.83 \times 10^2\ m/s} = \underline{2.1 \times 10^{-10}\ s}$$

22-37

(a) The resistance per unit length of a homogeneous conductor of conductivity σ and cross-sectional area a is given by $R/\ell = 1/\sigma a$. Therefore the inner wire has resistance per unit length

$$\frac{R_1}{\ell} = \frac{1}{\sigma_1 \pi r_1^2} \qquad (1)$$

while for the tubular conductor, we have

$$\frac{R_2}{\ell} = \frac{1}{\sigma_2 \pi (r_2^2 - r_1^2)} \qquad (2)$$

For the composite conductor the resistances add in parallel, so

$$\frac{\ell}{R} = \frac{\ell}{R_1} + \frac{\ell}{R_2} = \sigma_1 \pi r_1^2 + \sigma_2 \pi (r_2^2 - r_1^2)$$

The resistance per unit length is

$$\frac{R}{\ell} = \frac{1}{\pi [\sigma_1 r_1^2 + \sigma_2 (r_2^2 - r_1^2)]} \qquad (3)$$

(b) The voltage drop along the two conductors is the same, so we have

$$\frac{i_1 R_1}{\ell} = \frac{i_2 R_2}{\ell} \qquad (4)$$

Because the conductors are in parallel, the current is the sum of i_1 and i_2. Using this with eq. (4), we find

$$i = i_1 + i_2 = i_1 \left(1 + \frac{R_1}{R_2}\right)$$

$$= i_1 \left[1 + \frac{\sigma_2 (r_2^2 - r_1^2)}{\sigma_1 r_1^2}\right] \qquad (5)$$

Equation (5) yields

$$i_1 = \frac{i}{\left[1 + \frac{\sigma_2 (r_2^2 - r_1^2)}{\sigma_1 r_1^2}\right]}$$

....

$$= \frac{\sigma_1 r_1^2 i}{\sigma_1 r_1^2 + \sigma_2 (r_2^2 - r_1^2)} \tag{6}$$

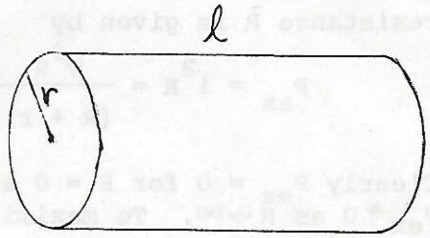
The current in the tubular conductor is easily found to be

$$i_2 = \frac{\sigma_2 (r_2^2 - r_1^2) i}{\sigma_1 r_1^2 + \sigma_2 (r_2^2 - r_1^2)} \tag{7}$$

(c) The power ratio is given by $P_2/P_1 = (i_2V_2)/(i_1V_1)$. Since the conductors are in parallel, $V_1 = V_2$, so $P_2/P_1 = i_2/i_1$, as was to be shown.

22-38

(a) We represent the filament as a cylinder of length ℓ and radius r, as shown at right. As pointed out in the exercise statement, in order that bulbs of various powers all exhibit the same filament temperature when operated at the same voltage, the bulbs are designed to have the same power per unit area of filament surface.



That is, P = KS where P is the power, S is the surface area of the filament, and K is a dimensional constant. The surface area is given by $S = 2\pi r \ell$, while the power

$$P = \frac{V^2}{R} = \frac{V^2 \sigma a}{\ell} = \frac{\sigma V^2 \pi r^2}{\ell} \tag{1}$$

Therefore the constant is given by

$$K = \frac{P}{S} = \frac{\sigma V^2 \pi r^2}{\ell (2\pi r \ell)} = \frac{\sigma V^2 r}{2\ell^2} \tag{2}$$

Since the bulbs are operated at the same voltage and the filaments have the same conductivity, eq. (2) implies that r/ℓ^2 must be constant: in fact, $r/\ell^2 = 2K/\sigma V^2$.

(b) According to eq. (1), the power ratio P_2/P_1 is given by

$$n \equiv \frac{P_2}{P_1} = \left(\frac{\sigma V^2 \pi r_2^2}{\ell_2}\right)\left(\frac{\ell_1}{\sigma V^2 \pi r_1^2}\right)$$

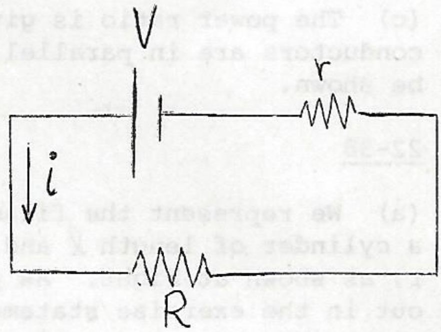
$$= \left(\frac{r_2^2}{r_1^2}\right)\left(\frac{\ell_1}{\ell_2}\right) \tag{3}$$

From part (a), we know that $r_2/\ell_2^2 = r_1/\ell_1^2$, or $r_2/r_1 = (\ell_2/\ell_1)^2$. Inserting this into eq. (3), we find

$$n = \left[\left(\frac{\ell_2}{\ell_1}\right)^2\right]^2 \left(\frac{\ell_1}{\ell_2}\right) = \left(\frac{\ell_2}{\ell_1}\right)^3$$

which implies that $\ell_2/\ell_1 = n^{1/3}$. The ratio of radii is $r_2/r_1 = \ell_2^2/\ell_1^2 = n^{2/3}$.

<u>22-39</u>



(a) The circuit is shown in the figure at the right. The current is given by $i = V/(R + r)$. The power delivered to the external resistance R is given by

$$P_{ex} = i^2R = \frac{V^2R}{(R + r)^2}$$

Clearly $P_{ex} = 0$ for $R = 0$ and $P_{ex} \to 0$ as $R \to \infty$. To maximize P_{ex}, we set

$$0 = \frac{dP_{ex}}{dR} = V^2\left[\frac{1}{(R + r)^2} - \frac{2R}{(R + r)^3}\right]$$

$$= R^2\left[\frac{r - R}{(R + r)^3}\right]$$

Therefore, the power delivered to the external resistance is maximized for $R = r$, as was to be shown.

(b) For $R = r$, the power delivered to the external resistance is

$$P_{ex} = \frac{V^2R}{(R + r)^2} = \frac{V^2r}{(r + r)^2} = \frac{V^2}{4r}$$

For the same external resistance value, the total power delivered by the battery is

$$P_{tot} = iV = \frac{V^2}{(R + r)} = \frac{V^2}{2r}$$

Therefore the efficiency is P_{ex}/P_{tot} = <u>50%</u> at maximum power. [NOTE: For a general value of R, the efficiency $P_{ex}/P_{tot} = R/(R + r)$.]

<u>22-40</u>

We let H denote the quantity of energy needed to bring the kettle to a boil. Then $H = P_1 t_1 = P_2 t_2$, where P_1 and P_2 are the powers of the two windings, and $H = V^2 t_1/R_1 = V^2 t_2/R_2$, where R_1 and R_2 are their resistances.

(a) When the windings are connected in series, they exhibit a resistance $R_s = R_1 + R_2$ and dissipate electrical energy at the rate

$$P_s = \frac{V^2}{R_s} = \frac{V^2}{(R_1 + R_2)}$$

The time required to bring the kettle to a boil is then

$$t_s = \frac{H}{P_s} = \frac{H}{V^2}(R_1 + R_2)$$

$$= \frac{HR_1}{V^2} + \frac{HR_2}{V^2} = t_1 + t_2$$

(b) When the windings are connected in parallel, each dissipates the power that it would if it alone were connected, so that

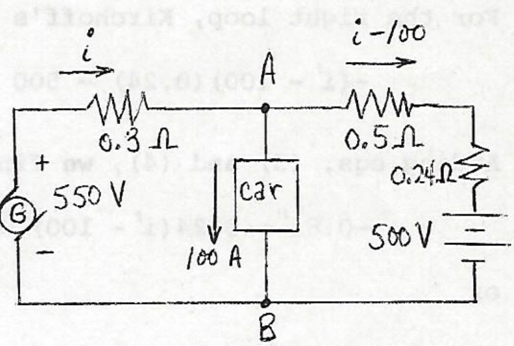
$$P_p = P_1 + P_2 = \frac{H}{t_1} + \frac{H}{t_2}$$

The time required to bring the kettle to a boil is therefore

$$t_p = \frac{H}{P_p} = \frac{H}{P_1 + P_2} = \frac{H}{[(H/t_1) + (H/t_2)]}$$

$$= \frac{t_1 t_2}{t_1 + t_2}$$

22-41



(a) The figure at the right indicates the situation when the trolley is 3.0 km from the generator end. In labeling the figure, we have applied Kirchoff's current law to the node A. (NOTE: We assume throughout the problem that the current through the trolley is 100 A, independent of the voltage drop. This may not be strictly true, but it is clearly the intent of the exercise statement that we make this assumption.) Applying Kirchoff's voltage law to the left loop, we obtain

$$-i(0.3) - (V_A - V_B) + 550 = 0 \qquad (1)$$

For the right loop, we find

$$-(i - 100)(0.5) - (i - 100)(0.24) - 500$$

$$+ (V_A - V_B) = 0 \qquad (2)$$

Adding eqs. (1) and (2) to eliminate $(V_A - V_B)$, we obtain

$$-0.3i - 0.74(i - 100) + 50 = 0$$

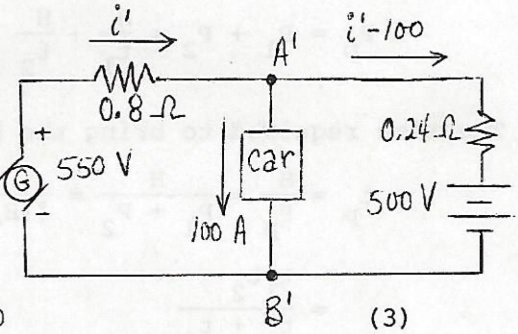
$$-1.04i = -124$$

$$i = 119.2 \text{ A}$$

The current supplied by the generator is 119.2 A while the current supplied to the battery is i - 100 = 19.2 A.

(b) Using eq. (1) with i = 119.2 A, we find $V_A - V_B$ = 550 - 0.3(119.2) = 514 V

(c) The figure shows the situation when the trolley is near the battery end of the line. In labeling the figure, we have applied Kirchoff's current law to the node A'. Applying Kirchoff's voltage law to the left loop, we find



$$-i'(0.8) - (V'_A - V'_B) + 550 = 0 \qquad (3)$$

For the right loop, Kirchoff's voltage law implies that

$$-(i' - 100)(0.24) - 500 + (V'_A - V'_B) = 0 \qquad (4)$$

Adding eqs. (3) and (4), we find

$$-0.8i' - 0.24(i' - 100) + 50 = 0$$

or

$$-1.04i' = -74$$

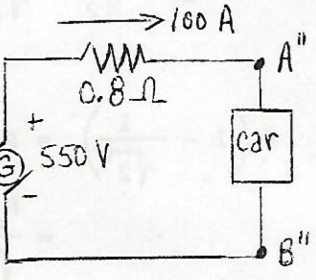
or

$$i' = 71.2 \text{ A}$$

The current supplied by the generator is 71.2 A. The current supplied to the battery is $i' - 100 = -28.8$ A; that is, the current supplied by the battery is 28.8 A.

(d) Using eq. (3) with $i' = 71.2$ A, we find $V_A' - V_B' = 550 - 0.8(71.2) = 493$ V.

(e) The figure depicts the situation when the trolley is at the end of the line and there is no battery in the circuit. Applying Kirchoff's voltage law around the circuit, we have



$$-(100)(0.8) - (V_A'' - V_B'') + 550 = 0$$

This implies that $V_A'' - V_B'' = 550 - 0.8(100) = 470$ V. Evidently, the purpose of the battery is to prevent the voltage across the trolley from getting too low.

22-42

We let i_1 denote the current in the circuit when the switch S is open. We let i_2 be the current in resistor R when the switch is closed. The effective resistance of the pair of identical resistors is $R_{eff} = R_o/2$ when the switch is closed, since the two resistors are in parallel. According to the exercise statement, we have

$$i_1^2 R_o = P = i_2^2 R_{eff} = i_2^2 R_o/2$$

so that

$$i_1 = i_2/\sqrt{2} \qquad (1)$$

Since the battery has constant voltage (call it V), the current with the switch S open is

$$i_1 = V/(r + R_o) \qquad (2)$$

and the current with S closed is

$$i_2 = V/(R + R_o/2) \qquad (3)$$

Equations (1) - (3) imply that

$$\frac{V}{(R + R_o)} = \frac{1}{\sqrt{2}} \frac{V}{(R + R_o/2)} \qquad (4)$$

Therefore we have

$$R + \frac{R_o}{2} = \frac{R}{\sqrt{2}} + \frac{R_o}{\sqrt{2}}$$

or

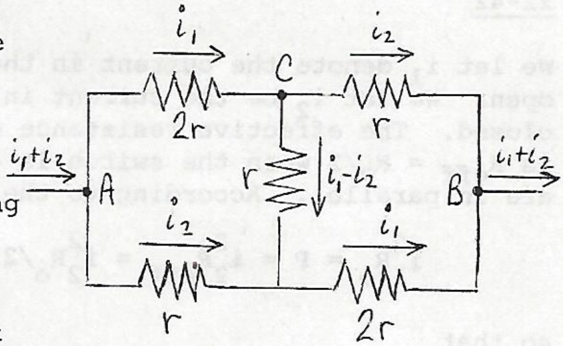
$$R\left(1 - \frac{1}{\sqrt{2}}\right) = R_o\left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right)$$

$$= \frac{R_o}{\sqrt{2}}\left(1 - \frac{1}{\sqrt{2}}\right)$$

so that $\underline{R = R_o/\sqrt{2}}$.

<u>22-43</u>

Symmetry considerations imply that the current in the upper right resistor must equal the current in the lower left resistor. (If the polarities of A and B were reversed, then the roles of those two resistors would be interchanged, and the directions of the currents would change.) Similarly, the currents in the two resistors 2r are equal. Using these implications of symmetry and applying Kirchoff's current law to nodes A, B, and C, we can label the currents in the circuit as shown at the right. Examining the path ACB, we see that the voltage drop is



$$V_A - V_B = i_1(2r) + i_2 r = 2i_1 r + i_2 r \qquad (1)$$

Applying Kirchoff's law to the left loop, we obtain

$$-i_1(2r) - (i_1 - i_2)r + i_2 r = 0$$

or

$$-3i_1 r + 2i_2 r = 0$$

or

$$i_1 = 2i_2/3 \qquad (2)$$

We use this to eliminate i_1 from eq. (1):

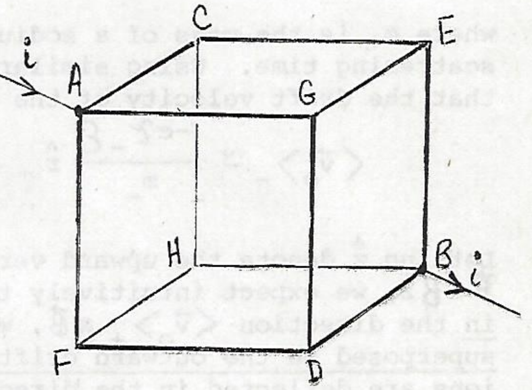
$$V_A - V_B = 2(2i_2/3)r + i_2 r = \frac{7}{3} i_2 r \qquad (3)$$

Using eqs. (2) and (3), we can now find the equivalent resistance of the circuit between A and B:

$$R_{AB} = \frac{V_A - V_B}{i_1 + i_2} = \frac{(7i_2 r/3)}{[(2i_2/3) + i_2]} = \underline{\frac{7r}{5}}$$

<u>22-44</u>

As can be seen in the figure at the right, the three wires (AC, AG, and AF) branching from A are symmetrically situated and therefore must carry equal currents. (After all, rotating the cube by 120° about the diagonal AB restores the original situation.) Clearly, the same is true of the wires EB, DB, and HB which carry current into B. Thus, if the total current flowing through the network from A to B is i, the currents in AC, AG, AF, EB, DB, and HB are all equal to i/3. The two wires CH and CE are symmetrically disposed with respect to AC, so the current in each must be $1/2(i_{AC}) = 1/2(i/3) = i/6$. Similar arguments show that the current is also i/6 in GE, GD, FD and FH. By examining the path ACEB, we conclude that when a current i is flowing from A to B, the voltage drop is



$$V_A - V_B = (i/3)r + (i/6)r + (i/3)r = \frac{5ir}{6}$$

Therefore the effective resistance between A and B is $R_{AB} = (V_A - V_B)/i = \underline{5r/6}$.