24-10

We use the symbols $\mathcal{M}$, L, T, and Q to represent the fundamental dimensions of mass, length, time, and electric charge, respectively. We use the symbol [x] to represent "the dimensions of the quantity x." From Eq. (24-48), magnetization M is measured in ampere meter^2/meter^3, so we have

$$[M] = L^{-1}T^{-1}Q \qquad (1)$$

According to Eq. (23-3) the magnetic field $\vec{\mathcal{B}}_o$ is measured in tesla, where

$$1 \text{ tesla} = \frac{1 \text{ newton sec}}{\text{coulomb meter}} = \frac{1 \text{ kg}}{C\cdot s}$$

Hence we find

$$[\mathcal{B}_o] = \mathcal{M}T^{-1}Q^{-1} \qquad (2)$$

The permeability of free space μ_o is defined by Eq. (23-38b) to be

$$\mu_o \equiv 4\pi \times 10^{-7} \text{ tesla}\cdot\text{meter/ampere}$$

$$= 4\pi \times 10^{-7} (\text{kg/C}\cdot\text{s})(\text{m})/(\text{C/s})$$

$$= 4\pi \times 10^{-7} \text{ kg}\cdot\text{m/C}^2$$

Therefore we have

$$[\mu_o] = \mathcal{M}LQ^{-2} \qquad (3)$$

Since the magnetic susceptibility is defined by $\chi \equiv \mu_o M/\mathcal{B}_o$, eqs. (1) - (3) imply that

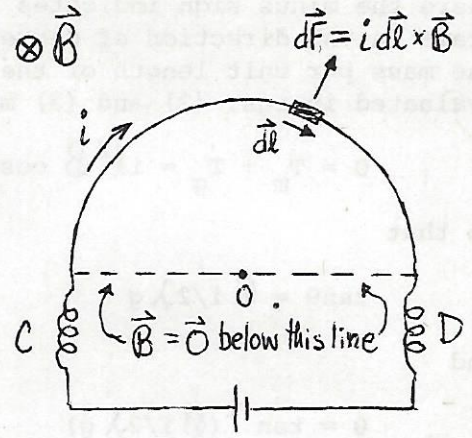
$$[\chi] \equiv \left[\frac{\mu_o M}{\mathcal{B}_o}\right] = \frac{[\mu_o][M]}{[\mathcal{B}_o]}$$

$$= \frac{(\mathcal{M}LQ^{-2})(L^{-1}T^{-1}Q)}{(\mathcal{M}T^{-1}Q^{-1})}$$

$$= \mathcal{M}^o L^o T^o Q^o = 1$$

That is, χ is a <u>dimensionless</u> quantity.

24-13



As shown in the figure at the right, there is a radially outward magnetic force $d\vec{F}$ on an element $d\vec{\ell}$ of the semicircle. The force has magnitude $dF = iBd\ell$. When these forces are summed over the entire semicircle, the components parallel to the diameter of the semicircle cancel. The net magnetic force is upward along the plane of the diagram, perpendicular to the diameter of the semicircle. The magnitude of the net force is $F_m = \int dF \sin\varphi$. Since $d\ell = rd\varphi$, we find $dF = iB\, d\ell = iBrd\varphi$, so that

$$F_m = \int_0^{\pi} iB r \sin\varphi\, d\varphi = iBr(-\cos\varphi\Big|_0^{\pi})$$

$$= iBr(-\cos\pi + \cos 0) = 2iBr$$

In order for the loop to be in equilibrium, the springs C and D must exert a force equal and opposite to $\vec{F}_m$, so the sum of the spring tensions is $2iBr$, as desired.

24-14

Referring to Fig. 24E-14, we note that the magnetic forces on the slanting sides are parallel to AC and therefore do not produce any torque about the axis AC. The magnetic force on the horizontal side of the frame is

$$\vec{F}_{mag} = i\vec{\ell}\times\vec{B} \qquad (1)$$

This force has magnitude $F_{mag} = i\ell B \sin 90° = i\ell B$ and points horizontally to the right in Fig. 24E-14, perpendicular to $\vec{\ell}$ and to $\vec{B}$. The magnetic torque about AC has magnitude

$$T_m = i\ell B(\ell\cos\theta) = i\ell^2 B\cos\theta \qquad (2)$$

and tends to increase the angle θ. The gravitational torque about the axis AC is

$$T_g = -[(\lambda\ell)g(\ell\sin\theta) + (\lambda\ell)g(\tfrac{1}{2}\ell\sin\theta)$$

$$+(\lambda\ell)g(\tfrac{1}{2}\ell\sin\theta)] = -2\lambda\ell^2 g\sin\theta \qquad (3)$$

where the minus sign indicates that gravity tends to turn the frame in the direction of decreasing θ. We have let λ denote the mass per unit length of the wire. In equilibrium, the torques evaluated in eqs. (2) and (3) must cancel, which implies

$$0 = T_m + T_g = i\ell^2 \mathcal{B} \cos\theta - 2\lambda\ell^2 g \sin\theta$$

so that

$$\tan\theta = \mathcal{B} i/2\lambda g$$

and

$$\theta = \tan^{-1}(\mathcal{B} i/2\lambda g)$$

With $\mathcal{B} = 1.0 \times 10^{-2}$ T, i = 10.0 A; $\lambda = 1.0$ g/cm = 0.10 kg/m, and $g = 9.80\ m/s^2$, we find $\tan\theta = (1.0 \times 10^{-2})(10.0)/[2(0.10) \times (9.80)] = 0.0510$, and $\theta = \tan^{-1}(0.0510) =$ <u>2.92°</u> = <u>0.0510 radians.</u>

<u>24-15</u>

Referring to Fig. 24E-15, we observe that the component of $\vec{\mathcal{B}}$ perpendicular to the plane of the loop is responsible for a force contribution on each current element $i d\vec{\ell}$. That contribution is directed toward the center of the loop. By pairing current elements located at opposite ends of a diameter, we see that these radially inward force contributions cancel. The net force on the loop is therefore due to the component of $\vec{\mathcal{B}}$ in the plane of the loop. This component, which has magnitude $\mathcal{B}\sin\theta$ and points radially outward at each point along the loop, produces an upward force of magnitude $dF = i d\ell\, \mathcal{B} \sin\theta$ on each current element. Hence, the net <u>upward</u> force on the loop has magnitude

$$F = \int dF = \int_{loop} i d\ell\, \mathcal{B} \sin\theta$$

$$= i\mathcal{B}\sin\theta \int_{loop} d\ell = \underline{2\pi R i \mathcal{B} \sin\theta}$$

<u>24-16</u>

(a) The number of protons per meter is the flux (number of protons passing per unit time) divided by the speed:

$$\frac{\Delta N}{\Delta x} = \frac{1}{v}\left(\frac{\Delta N}{\Delta t}\right) = \frac{1.00 \times 10^{15}\ s^{-1}}{5.00 \times 10^{7}\ m/s} = 2.00 \times 10^{7}\ m^{-1}$$

The charge per meter is therefore given by

$$\lambda \equiv \frac{\Delta Q}{\Delta x} = e\frac{\Delta N}{\Delta x} = (1.60 \times 10^{-19})(2.00 \times 10^{7})$$

$$= \underline{3.20 \times 10^{-12}\ C/m}$$

(b) In the rest frame of the protons, the distance between them is larger by a factor $(1 - v^2/c^2)^{-1/2}$. Since the charge per proton is unchanged, the charge per meter is reduced by the factor $(1 - v^2/c^2)^{1/2}$:

$$\lambda' = \lambda\left(1 - \frac{v^2}{c^2}\right)^{1/2}$$

$$= (3.20 \times 10^{-12})\left[1 - \left(\frac{5.00 \times 10^7}{3.00 \times 10^8}\right)^2\right]^{1/2}$$

$$= \underline{3.16 \times 10^{-12}\ \text{C/m}}$$

<u>24-17</u>

(a) From Example 24-3a, the charge per unit volume of mobile electrons is $-ne = -9.90 \times 10^9\ \text{C/m}^3$. Indexing the sections of the wire by 1, 2, and 3 (from left to right in Fig. 24E-17), we have $d_1 = 5.00 \times 10^{-4}$ m, $d_2 = 1.00 \times 10^{-3}$ m, and $d_3 = 1.50 \times 10^{-3}$ m. Since the cross-sectional area $a = \pi d^2/4$, we find $a_1 = 1.963 \times 10^{-7}\ \text{m}^2$, $a_2 = 7.854 \times 10^{-7}\ \text{m}^2$, and $a_3 = 1.767 \times 10^{-6}\ \text{m}^2$. The current i is given by $i = ne|v_d|a$, where $|v_d|$ is the drift speed, so $|v_d| = i/nea$. With i = 1.00 A we find

$$|v_{d1}| = \frac{i}{nea_1} = \frac{1.00}{(9.90 \times 10^9)(1.963 \times 10^{-7})} = 5.15 \times 10^{-4}\ \text{m/s}$$

$$|v_{d2}| = \frac{i}{nea_2} = \frac{1.00}{(9.90 \times 10^9)(7.854 \times 10^{-7})} = 1.29 \times 10^{-4}\ \text{m/s}$$

and

$$|v_{d3}| = \frac{i}{nea_3} = \frac{1.00}{(9.90 \times 10^9)(1.76 \times 10^{-6})} = 5.72 \times 10^{-5}\ \text{m/s}$$

Since these speeds [and the other speeds that arise in part (b)] are tiny compared with the speed of light, we can neglect special-relativistic corrections (which are of order v^2/c^2).

(b) Using the convention that velocities to the right are positive, the electron drift velocities v_{d1}, v_{d2}, and v_{d3} are all negative, since positive current is flowing to the right. If we adopt the reference frame of an observer moving to the right at speed $v_o \ll c$, the velocities of the positive and negative charge carriers are $v'_+ = -v_o$ and $v'_- = v_d - v_o$. Using $v_o = 9.40 \times 10^{-5}$ m/s, we find $v'_{+1} = v'_{+2} = v'_{+3} = -9.40 \times 10^{-5}$ m/s, $v'_{-1} = -6.09 \times 10^{-4}$ m/s, $v'_{-2} = -2.23 \times 10^{-4}$ m/s, and $v'_{-3} = -1.51 \times 10^{-4}$ m/s. In each part of the wire, the current due to the conduction electrons is given by $i'_- = -nev'_-a$, while the current due to the positive ions is $i'_+ = nev'_+a$. Inserting numerical values, we find

$$i'_- = -nev'_{-1}a_1 = (-9.90 \times 10^9)(-6.09 \times 10^{-4})(1.963 \times 10^{-7})$$
$$= 1.18\ \text{A}$$

$$i'_{+1} = nev'_{+1}a_1 = (9.90 \times 10^9)(-9.40 \times 10^{-5})(1.963 \times 10^{-7})$$
$$= -0.18\ \text{A}$$

Continuing with the numerical evaluations, we obtain

$$i'_{-2} = -nev'_{-2}a_2 = (-9.90 \times 10^9)(-2.23 \times 10^{-4})(7.854 \times 10^{-7})$$
$$= 1.73\ \text{A}$$

$$i'_{+2} = nev'_{+2}a_2 = (9.90 \times 10^9)(-9.40 \times 10^{-5})(7.854 \times 10^{-7})$$
$$= -0.73\ \text{A}$$

$$i'_{-3} = -nev'_{-3}a_3 = (-9.90 \times 10^9)(-1.51 \times 10^{-4})(1.767 \times 10^{-6})$$
$$= 2.64\ \text{A}$$

$$i'_{+3} = nev'_{+3}a_3 = (9.90 \times 10^9)(-9.40 \times 10^{-5})(1.767 \times 10^{-6})$$
$$= -1.64\ \text{A}$$

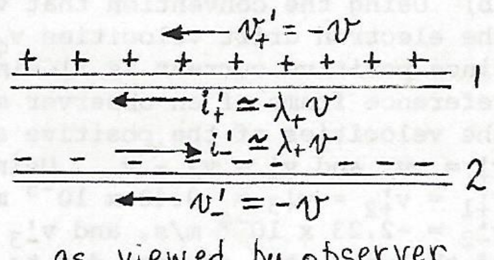
Note that the total current in each section is

$$i' \equiv i'_{-k} + i'_{+k} = ne(v'_{+k} - v'_{-k})a_k = ne(v_{+k} - v_{-k})a_k$$
$$= i = 1.00\ \text{A}$$

24-18

(a) The electric field produced by wire 1 at the location of wire 2 has magnitude $\mathcal{E}_+ = \lambda_+/2\pi\epsilon_0 r$. The Lorentz force per unit length on wire 2 is $(F_e/L) = \lambda_-\mathcal{E}_+ = -\lambda_+^2/2\pi\epsilon_0 r$. The minus sign indicates that wire 2 is attracted by wire 1.

(b) For an observer moving rightward at a slow speed ($v \ll c$), the charge densities of the wires are not significantly altered. Furthermore, the separation r' equals the rest-frame separation r. Hence the electric force per unit length $(\vec{F}'_e/L')$ equals $(\vec{F}_e/L)$ to order v/c. However, as indicated



as viewed by observer moving rightward at speed v in the figure on the previous page, there are two antiparallel currents: $i'_+ = \lambda'_+(-v) \simeq -\lambda_+ v$ in wire 1, and $i'_- = (\lambda'_-)(-v) \simeq \lambda_+ v$ in wire 2. The two wires repel each other with a magnetic force per unit length

$$\left|\frac{\vec{F}'_M}{L'}\right| = \frac{\mu_o |i'_+ i'_-|}{2\pi r'} \simeq \frac{\mu_o \lambda_+^2 v^2}{2\pi r} \qquad (1)$$

This repulsive force is much smaller than the attractive force $|\vec{F}'_e/L'| \simeq \lambda_+^2/2\pi\epsilon_o r$. In the primed frame, <u>the force between the wires includes both electric and magnetic contributions.</u> On the basis of eq. (1), we might expect that the overall Lorentz force per unit length on wire 2 is

$$\frac{\vec{F}'}{L'} \simeq \frac{\vec{F}'_e}{L} + \frac{\vec{F}'_M}{L} = \frac{\lambda_+^2}{2\pi\epsilon_o r}(1 - \mu_o\epsilon_o v^2)$$

$$= \frac{\lambda_+^2}{2\pi\epsilon_o r}\left(1 - \frac{v^2}{c^2}\right) \qquad (2)$$

toward wire 1. (<u>But see below!</u>) Here we have used $\mu_o\epsilon_o \equiv 1/c^2$, as stated in Eq. (24-22). Notice for $v \ll c$, the electric force is much stronger than the magnetic force. Even for high speeds, <u>there is no frame in which the force is solely magnetic.</u> For speeds comparable to c, the charge densities per unit length are increased due to Lorentz contraction while the separation r' remains equal to r, so $|\vec{F}'_e|/L'$ certainly does <u>not</u> approach zero as v c.
<u>NOTE:</u> Since the magnetic force per unit length given in eq. (1) is of order v^2/c^2 compared to the electric field, expression (2) for the Lorentz force is really not self-consistent. When fully relativistic expressions are used, we find that the charges per unit length are given by

$$\lambda'_- = -\lambda'_+ = -\lambda_+\left(1 - \frac{v^2}{c^2}\right)^{-1/2} \qquad (3)$$

so that the currents are

$$i'_+ = -i'_- = -\lambda'_+ v = -\lambda_+ \cdot \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \cdot v \qquad (4)$$

Then the exact electric force per unit length on wire 2 is

$$\frac{\vec{F}'_e}{L'} = \frac{\lambda'_+\lambda'_-}{2\pi\epsilon_o r'} = \frac{-\lambda_+^2}{2\pi\epsilon_o r}\left(1 - \frac{v^2}{c^2}\right)^{-1} \qquad (5)$$

where the minus sign indicates attraction. The magnetic force per unit length between the wires carrying antiparallel currents is repulsive and has magnitude

$$\frac{|\vec{F}'_M|}{L'} = \frac{\mu_0 |i'_+ i'_-|}{2\pi r'} = \frac{\lambda_+^2}{2\pi \epsilon_0 r}\left(1 - \frac{v^2}{c^2}\right)^{-1} \frac{v^2}{c^2} \qquad (6)$$

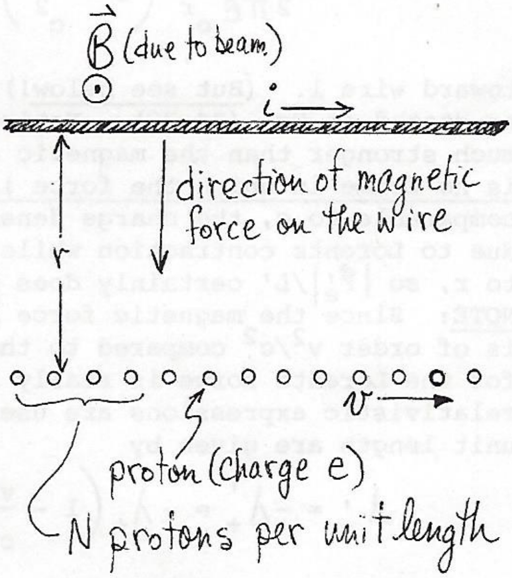
Hence the exact Lorentz force per unit length is an attraction of magnitude

$$\frac{|\vec{F}'|}{L'} = \frac{|\vec{F}'_e|}{L'} - \frac{|\vec{F}'_M|}{L'} = \frac{\lambda_+^2}{2\pi \epsilon_0 r}\left(1 - \frac{v^2}{c^2}\right)^{-1} \cdot \left(1 - \frac{v^2}{c^2}\right)$$

$$= \frac{\lambda_+^2}{2\pi \epsilon_0 r}$$

<u>That is, the Lorentz force per unit length has the same value in both frames.</u>

<u>24-19</u>



(a) For definiteness, we assume that both the proton beam and the current in the wire are directed to the right with the wire located above the beam, as indicated in the figure. Then the magnetic field produced by the protons at the location of the wire is directed up out of the plane of the figure and has magnitude $B = \mu_0 Nev/2\pi r$. Since the wire is electrically neutral, it only experiences a magnetic force. The Lorentz force per unit length on the wire is directed toward the beam and has magnitude

$$\frac{F}{L} = iB = \frac{\mu_0 Nevi}{2\pi r} \qquad (1)$$

[NOTE: If the beam current and the current in the wire are assumed to be <u>antiparallel</u>, the force per unit length is still given by eq. (1), but it is a <u>repulsive</u> force.]

(b) We use $N = 1.00 \times 10^6\ m^{-1}$ and $v = 2.00 \times 10^6$ m/s, which correspond to a proton energy of approximately 21,000 electron-volts and a beam current of approximately 0.32 μA. We also assume $i = 10.0$ A and $r = 5.00 \times 10^{-2}$ m. Then, according to eq. (1), the magnitude of the force per unit length on the wire is

$$\frac{F}{L} = \frac{(4\pi \times 10^{-7})(1.00 \times 10^{6})(1.60 \times 10^{-19})(2.00 \times 10^{6})(10.0)}{2\pi(5.00 \times 10^{-2})}$$

$$= \underline{1.28 \times 10^{-11} \text{ N/m}}$$

(c) An observer at rest with respect to the wire interprets the force as being entirely magnetic, as detailed in parts (a) and (b). An observer moving with the beam of protons does not observe a magnetic field produced by the protons. She observes only an electric field $\vec{\mathcal{E}}'$ produced by the protons. In addition, she observes the wire to be charged, where the linear charge density $\lambda' \neq 0$ arises from the differing Lorentz contraction effects on the densities of the positive and negative charges in the wire (due to their differing motions.) The Lorentz force which she observes is entirely electric.

(d) As described in part (c), an observer moving along with the beam measures a force per unit length on the wire given by

$$\frac{\vec{F}'}{L'} = \lambda' \vec{\mathcal{E}}' \qquad (2)$$

where $\vec{\mathcal{E}}'$ is the electric field produced by the proton beam and λ' is the observed linear charge density of the wire.

24-20

(a) Referring to Fig. 24-11, we calculate the torque $\vec{T}'$ on the bar magnet, using an origin O' at the north pole of the magnet:

$$\vec{T}' = \vec{r}'_+ \times \vec{F}'_+ + \vec{r}'_- \times \vec{F}'_- \qquad (1)$$

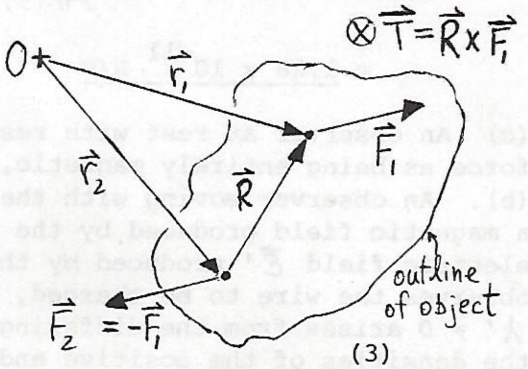
Here $\vec{r}'_+$ and $\vec{r}'_-$ are the radius vectors of the poles, as measured from O'. We have $\vec{r}'_+ = \vec{0}$ and $\vec{r}'_- = \vec{d}_- - \vec{d}_+ = -2\vec{d}_+$. The forces $\vec{F}'_+$ and $\vec{F}'_-$ are given by $\vec{F}'_+ = |\psi|\vec{\mathcal{B}} = \vec{F}_+$ and $\vec{F}'_- = -|\psi|\vec{\mathcal{B}} = \vec{F}_-$. Thus eq. (1) becomes

$$\vec{T}' = \vec{0} \times (|\psi|\vec{\mathcal{B}}) + (-2\vec{d}_+) \times (-|\psi|\vec{\mathcal{B}})$$

$$= (2|\psi|\vec{d}_+) \times \vec{\mathcal{B}} = \vec{m} \times \vec{\mathcal{B}} \qquad (2)$$

Equation (2) shows that the vector $\vec{T}'$ is identical with $\vec{T}$, the torque evaluated in Sec. 24-3 using an origin at the center of the dipole. In fact, for any choice of origin O", the torque $\vec{T}''$ can be shown to be $\vec{m} \times \vec{\mathcal{B}}$. [A generalization of this result to all equal and opposite force pairs is given in part (b).] In Sec. 21-4, the torque on an electric dipole $\vec{p}$ in a uniform electric field was shown to be $\vec{p} \times \vec{\mathcal{E}}$. The form of this expression is analogous to the expression $\vec{m} \times \vec{\mathcal{B}}$ obtained in eq. (2), provided that we let the magnetic dipole $\vec{m}$ correspond to the electric dipole $\vec{p}$, and the magnetic field $\vec{B}$ correspond to the electric field $\vec{\mathcal{E}}$.

(b) We suppose that an object is subject to forces $\vec{F}_1$ and $\vec{F}_2 = -\vec{F}_1$, and that the point of application of $\vec{F}_1$ is displaced by $\vec{R}$ from the point of application of $\vec{F}_2$. This is shown in the figure at the right. Using an arbitrary origin O, the torque produced by the couple $(\vec{F}_1, \vec{F}_2)$ is



$$\vec{T} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 \tag{3}$$

where $\vec{r}_1$ and $\vec{r}_2$ are the vectors from O to the points of application, as shown. Since $\vec{F}_2 = -\vec{F}_1$ and $\vec{r}_1 - \vec{r}_2 = \vec{R}$, eq. (3) can be rewritten as

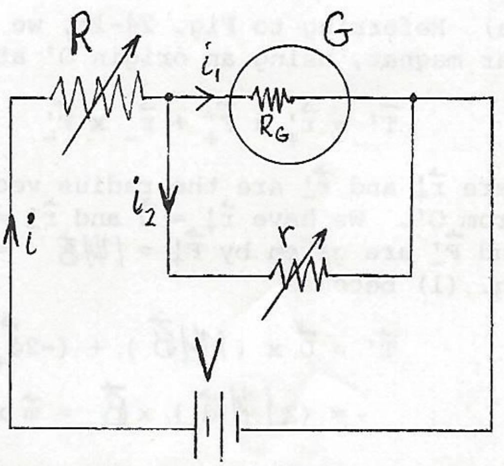
$$\vec{T} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times (-\vec{F}_1) = (\vec{r}_1 - \vec{r}_2) \times \vec{F}_1$$

$$= \vec{R} \times \vec{F}_1 \tag{4}$$

This last expression is clearly <u>independent of the choice of origin</u>.

24-21

The circuit is indicated schematically at the right. Since the low variable resistance r has a marked effect on the current through the galvanometer, r must be of the same order of magnitude as R_G, the unknown galvanometer coil resistance. Since the variable high resistance R is much larger than either R_G or r, the total resistance of the circuit is not significantly different from R and the current i flowing from the battery is essentially given by $i \simeq V/R$, regardless of the value of r or R_G.



Since the galvanometer and r are in parallel, we have $i_1 + i_2 = i$ and $i_1 R_G = i_2 r$. Hence when the low variable resistance r has been adjusted until $i_1 = i_2 = i/2$, we have $R_g = r$, as was to be shown.

24-22

When the switch is in position D, the entire shunt resistance $R_S = R_{AB} + R_{BC} + R_{CD}$ is in parallel with the galvanometer resistance R_G. Therefore the voltage drop across ABCD equals the voltage drop across the galvanometer:

$$i_G R_G = i_S R_S \qquad (1)$$

Since the total current entering the ammeter is $i_D = i_G + i_S$, we can use eq. (1) to find that

$$i_D = i_G\left(\frac{R_G + R_S}{R_S}\right) \qquad (2)$$

Since $R_S = R_{AB} + R_{BC} + R_{CD} = (0.01 + 0.09 + 0.90) = 1.00\ \Omega$ and $R_G = 99.0\ \Omega$, we have $i_D = 100\ i_G$. Since the full-scale galvanometer current $(i_G)_{max} = 1.00 \times 10^{-3}$ A, the maximum measurable current with the switch in position D is $(i_D)_{max} = 100\ (1.00 \times 10^{-3})$ = 0.100 A.

With the switch in position C, the shunt resistance is $R'_S = R_{AB} + R_{BC} = (0.010 + 0.090) = 0.100\ \Omega$. Since the resistance R_{CD} is now in series with R_G, the effective galvanometer resistance is $R'_G = R_G + R_{CD} = (99.0 + 0.9) = 99.9\ \Omega$. Since R'_S and R'_G are in parallel, eq. (2) can be generalized to yield

$$i_C = i'_G\left[\frac{(R'_G + R'_S)}{R'_S}\right] \qquad (3)$$

Since $(i'_G)_{max} = (i_G)_{max} = 1.00 \times 10^{-3}$ A, we have $(i_C)_{max} = (1.00 \times 10^{-3})(99.9 + 0.100)/(0.100) = $ 1.00 A. Finally, with the shunt in position B, a shunt resistance $R''_S = R_{AB} = 0.0100\ \Omega$ is in parallel with an effective galvanometer resistance $R''_G = R_G + R_{CD} + R_{BC} = (99.0 + 0.9 + 0.09) = 99.99\ \Omega$. Then

$$i_B = i''_G\left[\frac{(R''_G + R''_S)}{R''_S}\right] \qquad (4)$$

which gives $(i_B)_{max} = (1.00 \times 10^{-3})(99.99 + 0.01)/(0.01) = $ 10.0 A.

24-23

(a) The magnetic moment of the coil is N times the magnetic moment of each loop: $\vec{m} = Ni\vec{a}$. Here the "area vector" $\vec{a}$ is directed vertically upward in Fig. 24-17. According to Eq. (24-37c), the vector torque on the coil is $\vec{T} = \vec{m} \times \vec{\mathcal{B}} = Ni(\vec{a} \times \vec{\mathcal{B}})$. This vector has magnitude $T = Nia\mathcal{B}\sin\theta$, where θ is the angle between $\vec{\mathcal{B}}$ and $\vec{m}$.

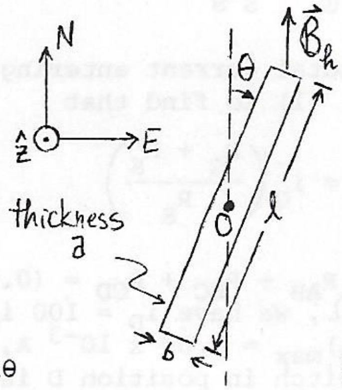
(b) The work done by this torque in each half-revolution is

$$\frac{1}{2}\Delta W = \int_0^{\pi} T(\theta)\,d\theta = Nia\mathcal{B}\int_0^{\pi} \sin\theta\, d\theta$$

$$= Nia\mathcal{B}\left(-\cos\theta\Big|_0^{\pi}\right) = 2Nia\mathcal{B}$$

Hence the work output per revolution is $\Delta W = 4Nia\mathcal{B}$. If there are ν revolutions per unit time, the power output $P = dW/dt = \nu\Delta W = \underline{4\nu Nia\mathcal{B}}$.

24-24



The figure at the right shows the bar magnet as viewed from above, with north upward (and hence east to the right!). The vertical component of the torque on the magnet is

$$T_z = (\vec{m} \times \vec{\mathcal{B}})_z = -m\mathcal{B}_h \sin\theta \qquad (1)$$

The orientation $\theta(t)$ of the magnet is governed by the rotational form of Newton's second law:

$$I_o \frac{d^2\theta}{dt^2} = T_z \qquad (2)$$

where I_o is the moment of inertia of the magnet about a vertical axis through the midpoint O. Referring to Table 10-1, we have

$$I_o = \frac{1}{12}M(\ell^2 + b^2) = \frac{1}{12}\rho\ell ab(\ell^2 + b^2) \qquad (3)$$

where M, ρ, and a are the mass, mass density, and (vertical) thickness of the magnet. Assuming that the magnet is suspended "flat", rather than "on edge", we have $\ell = 0.10$ m, $b = 0.010$ m, and $a = 0.0050$ m. With $\rho = 7.9 \times 10^3$ kg/m^3, eq. (3) yields

$$I_o = \frac{1}{12}(7.9 \times 10^3)(0.10)(0.010)(0.0050)[(0.10)^2 + (0.010)^2]$$

$$= 3.32 \times 10^{-5} \text{ kg m}^2 \qquad (4)$$

Equations (1) and (2) imply that

$$\frac{d^2\theta}{dt^2} = -\frac{m\mathcal{B}_h}{I_o}\sin\theta \qquad (5)$$

For large-amplitude oscillations, the period of the motion governed by eq. (5) is amplitude-dependent. However, assuming $\theta \ll 1$ radian, eq. (5) can be replaced by

$$\frac{d^2\theta}{dt^2} \simeq - \frac{m\mathcal{B}_h}{I_o}\theta \qquad (6)$$

which is the differential equation for harmonic motion with period

$$t_o = 2\pi\sqrt{\frac{I_o}{m\mathcal{B}_h}} \qquad (7)$$

Solving eq. (7) for the dipole moment, we obtain

$$m = \frac{\mathcal{B}_h t_o^2}{4\pi^2 I_o} \qquad (8)$$

Using $\mathcal{B}_h = 2 \times 10^{-5}$ T, $t_o = 20$ s, and I_o from eq. (4), we find

$$m = \frac{(2 \times 10^{-5})(20)^2}{4\pi^2(3.32 \times 10^{-5})} = \underline{6.1\ \text{A}\cdot\text{m}^2}$$

24-25

The magnetic moment of a bar magnet of length 2d has magnitude $m = 2|\psi|\cdot d$, where $|\psi|$ is the pole strength. In terms of the magnetization $\vec{M}$, the magnetic moment of the bar magent is $\vec{m} = \vec{M}(2ad)$, where a is the cross-sectional area. Hence $|\psi| = m/2d = aM$. With $a = 1.0\ \text{cm}^2 = 1.00 \times 10^{-4}\ \text{m}^2$ and $M = 1.00 \times 10^2$ A/m, we obtain $|\psi| = (1.00 \times 10^{-4})(1.00 \times 10^2) = \underline{1.00 \times 10^{-2}\ \text{A}\cdot\text{m}}$.

24-26

(a) According to Eq. (23-65), the externally-applied field along the axis of the toroid is $\mathcal{B}_o = \mu_o ni$. With $n = 1000\ \text{m}^{-1}$ and $i = 1.0$ A, we find $\mathcal{B}_o = (4\pi \times 10^{-7})(1000)(1.0) = 1.257 \times 10^{-3}$ T $= \underline{1.26 \times 10^{-3}\ \text{T}}$.

(b) The "demagnetizing field" $\vec{\mathcal{B}}_d = \vec{\mathcal{B}}_{int} - \vec{\mathcal{B}}_o$. Since $\vec{\mathcal{B}}_o$ and $\vec{\mathcal{B}}_{int}$ are parallel, the magnitude $\mathcal{B}_d$ is given by $\mathcal{B}_d = \mathcal{B}_{int} - \mathcal{B}_o = 0.60\ \text{T} - 1.257 \times 10^{-3}\ \text{T} = \underline{0.599\ \text{T}}$.

(c) We solve Eq. (24-50a) for the amperean surface current to obtain

$$\frac{i_s}{\ell} = \frac{\mathcal{B}_d}{\mu_o} = \frac{0.5987}{4\pi \times 10^{-7}} = \underline{4.76 \times 10^5\ \text{A/m}}$$

(d) In order to have $\mathcal{B}_o' = \mu_o ni' = 0.60$ T, we would need

$$i' = \frac{\mathcal{B}_o'}{\mu_o n} = \frac{0.600}{(4\pi \times 10^{-7})(10^3)} = \underline{477\ \text{A}}$$

(e) The relative permeability $K_m = \mathcal{B}_{int}/\mathcal{B}_o$, so with $\mathcal{B}_{int} = 0.60$ T and $\mathcal{B}_o = 1.257 \times 10^{-3}$ T, we obtain K = 477.

24-27

According to Curie's law, the susceptibility $\chi = C/T$, so that $C = \chi T$. Using the line fitted to the data in Fig. 24-25, we see that $\chi = 0.50$ for $1/T = 8.0 \times 10^{-3}\ K^{-1}$, or for T = 125 K. According to these numerical values, the Curie constant for chromium potassium alum is C = (0.50)(125) = 63 K.

24-28

According to the Curie-Weiss law, $\chi = C/(T - T_c)$, so that

$$\frac{1}{\chi} = \frac{T - T_c}{C} \qquad (1)$$

Hence T_c is the T-intercept of the best-fit straight line in Fig. 24-27. As noted in the caption, this yields T_c = 310 K. Furthermore, we find that $1/\chi = 26.8$ for T = 1400 K and that $1/\chi = 2.0$ for T = 400 K. Since eq. (1) implies that

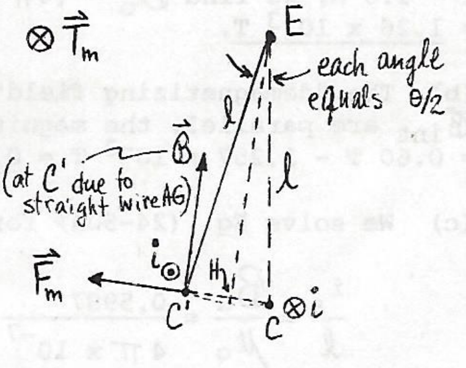
$$\frac{\Delta(1/\chi)}{\Delta(T)} = \frac{1}{C} \qquad (2)$$

we find

$$C = \frac{\Delta(T)}{\Delta(1/\chi)} = \frac{(1400 - 400)\ K}{(26.8 - 2.0)}$$

$$= 40.3\ K \simeq 40\ K$$

24-29



(a) The situation is shown in end view in the figure at the right. The magnetic torque on the frame about E is due to the force $\vec{F}_m$ on the side C'D'. The lever arm is $\overline{EH}$, so the torque about EF has magnitude $T_m = F_m \times \overline{EH}$. The geometry of triangle EHC shows that $\overline{EH} = \ell \cos(\theta/2)$, so that

$$T_m = F_m \ell \cos(\theta/2) \simeq F_m \ell \qquad (1)$$

Here we have used the fact that $\theta \ll 1$ radian. Notice that the vector torque $\vec{T}_m$ is directed from E toward F. That is, it tends to turn the frame clockwise as viewed from A.

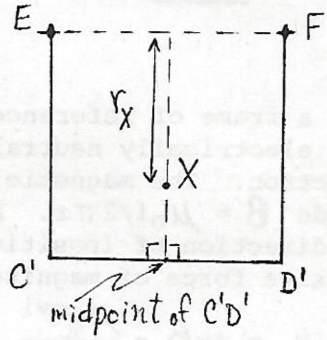
(b) Applying the results of Sec. 24-1, we find that the magnetic force on C'D' is

$$F_m = \frac{\mu_0}{2\pi}\frac{i^2}{\overline{CC'}}\overline{C'D'} \qquad (2)$$

But $\overline{CC'} = 2\ell\sin(\theta/2) \simeq \ell\theta$ since $\theta \ll 1$ radian. Since $\overline{C'D'} = \ell$ and $\mu_0/2\pi = 2 \times 10^{-7}\ N/A^2$, eq. (2) becomes

$$F_m = (2 \times 10^{-7}\ N/A^2)i^2/\theta \qquad (3)$$

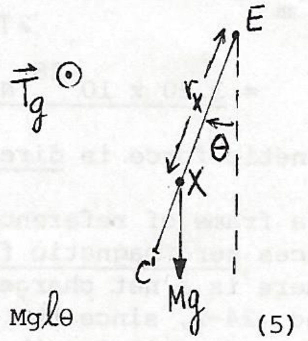
(c) In the figure at the right, we show the frame EC'D'F. We let X denote the center of mass and r_X denote its distance from the axis EF. By symmetry, X must be located along the perpendicular bisector of C'D'. From the definition of the center of mass, the distance r_X satisfies the equation.



$$3mr_X = m\left(\frac{\ell}{2}\right) + m\ell + m\left(\frac{\ell}{2}\right) \qquad (4)$$

Here m is the mass of each of the sides EC', C'D', and D'F. Solving eq. (4) for r_X, we obtain $r_X = \frac{2}{3}\ell$ as was to be shown.

(d) As discussed in Chapter 10, the gravitational torque on the frame is the product of the gravitational force (which effectively acts at the center of mass X) and its lever arm. Referring to the figure at the right, the gravitational torque about EF has magnitude



$$T_g = Mgr_X\sin\theta = \frac{2}{3}Mg\ell\sin\theta \simeq \frac{2}{3}Mg\ell\theta \qquad (5)$$

Notice that this is a restoring torque: its direction is opposite to that of the magentic torque.

(e) In equilibrium, the torques must cancel: $T_m = T_g$. Using eqs. (1), (3), and (5), we have

$$\frac{2}{3}Mg\ell\theta = F_m\ell = (2 \times 10^{-7}\ N/A^2)i^2/\theta$$

so that

$$i^2 = \frac{(10^7\ A^2/N)Mg\theta^2}{3}$$

or

$$i = \theta\sqrt{\frac{(10^7\ A^2/N)Mg}{3}} \qquad (6)$$

With $\theta = 0.023$ rad, $M = 0.022$ kg, and $g = 9.80\ m/s^2$, we find

$$i = (0.023)\cdot\sqrt{\frac{(10^7)(0.022)(9.80)}{3}}$$

$$= \underline{19.5\ A}$$

24-30

(a) In a frame of reference at rest with respect to the wire, the wire is electrically neutral, so there is zero electric force on the electron. The magnetic field a distance r from the wire has magnitude $\mathcal{B} = \mu_0 i/2\pi r$. An electron moving at speed v parallel to the direction of (positive) current flow in the wire experiences a repulsive force of magnitude

$$F_m = ev\mathcal{B} = \frac{\mu_0 evi}{2\pi r} \qquad (1)$$

With $i = 0.100$ A, $v = 1.00 \times 10^{-3}$ m/s, and $r = 1.00 \times 10^{-2}$ m, we find

$$F_m = \frac{(4\pi \times 10^{-7})(1.60 \times 10^{-19})(1.00 \times 10^{-3})(0.100)}{2\pi(1.00 \times 10^{-2})}$$

$$= \underline{3.20 \times 10^{-28}\ N}$$

This magnetic force is directed away from the wire.

(b) In a frame of reference moving with the electron, the electron experiences zero magnetic force since it is stationary. In this frame there is a net charge density on the current-carrying wire. As in Sec. 24-2, since the drift speed v is much less than c, the net charge per unit length is $\lambda' = -\lambda_+ v^2/c^2$, where $i = \lambda_+ v$. Hence the charge per unit length is $\lambda' = -(i/v)v^2/c^2 = -iv/c^2$. As a result there is a repulsive electric force on the electron of magnitude

$$F'_e = e\mathcal{E}' = \frac{e(iv/c^2)}{2\pi\epsilon_0 r} = \frac{\mu_0 evi}{2\pi r} \qquad (2)$$

Equation (2), in which the fractional error is of order v^2/c^2, agrees with the (exact) expression (1). Hence, to this order of accuracy, the electric force is 3.20×10^{-28} N, away from the wire.

24-31

(a) In the laboratory frame, the electric field $\vec{\mathcal{E}}$ points away from the beam. With a charge per unit length $\lambda = 2.00 \times 10^{-12}$ C/m and a beam-to-proton distance r = 0.0100 m, the field magnitude is

$$\mathcal{E} = \frac{\lambda}{2\pi\epsilon_o r} = \frac{2\lambda}{4\pi\epsilon_o r}$$

$$= \frac{2 \times (8.99 \times 10^9)(2.00 \times 10^{-12})}{(1.00 \times 10^{-2})}$$

$$= 3.596 \text{ N/C}$$

Hence the electric force on the proton is $\vec{F}_e = e\vec{\mathcal{E}} = (1.60 \times 10^{-19}) \times (3.596) = \underline{5.75 \times 10^{-19}}$ N away from the beam. The magnetic field lines circle the beam. Since the current carried by the beam is $i = \lambda v_p = (2.00 \times 10^{-12})(5.00 \times 10^7) = 1.00 \times 10^{-4}$ A, the field strength at the location of the proton is

$$\mathcal{B} = \frac{\mu_o i}{2\pi r} = \frac{(2 \times 10^{-7})(1.00 \times 10^{-4})}{(1.00 \times 10^{-2})} = 2.00 \times 10^{-9} \text{ T}$$

Since the proton's velocity $\vec{v}_p$ is perpendicular to $\vec{\mathcal{B}}$, the magnetic force $\vec{F}_m = e\vec{v}_p \times \vec{\mathcal{B}}$ has magnitude

$$F_m = ev_p \mathcal{B} = (1.60 \times 10^{-19})(5.00 \times 10^7)(2.00 \times 10^{-9})$$

$$= \underline{1.60 \times 10^{-20} \text{ N}}$$

This force is directed toward the beam.

(b) In the proton's rest frame, the beam's charge per unit length is $\lambda' = \lambda\sqrt{1 - v_p^2/c^2}$, since the beam particles are also at rest in this frame:

$$\lambda' = (2.00 \times 10^{-12})\sqrt{1 - \left(\frac{5.00 \times 10^7}{3.00 \times 10^8}\right)^2}$$

$$= 1.972 \times 10^{-12} \text{ C/m}$$

The beam-to-proton distance r' = r, since transverse distances are unaffected by the Lorentz transformation. Hence the electric field due to the beam at the position of the proton is

$$\mathcal{E}' = \frac{\lambda'}{2\pi\epsilon_o r'} = \frac{2\lambda'}{4\pi\epsilon_o r} = \frac{2(8.99 \times 10^9)(1.972 \times 10^{-12})}{(1.00 \times 10^{-2})}$$

$$= 3.546 \text{ N/C}$$

Hence the electric force on the proton is $F'_e = e\mathcal{E}'$ $= (1.60 \times 10^{-19})(3.546) = 5.67 \times 10^{-19}$ N away from the beam. The magnetic force $\vec{F}'_m$ is evidently zero in the proton's rest frame.

(c) To find the (common) velocity v''_p of the protons in a frame of reference moving at speed $V = 1.00 \times 10^8$ m/s in the direction of the beam, we use the Lorentz velocity transformation:

$$v''_p = \frac{v_p - V}{1 - v_p V/c^2} = \frac{5.00 \times 10^7 - 1.00 \times 10^8}{1 - \frac{(5.00 \times 10^7)(1.00 \times 10^8)}{(3.00 \times 10^8)^2}}$$

$$= -5.294 \times 10^7 \text{ m/s}$$

Due to the Lorentz contraction, the beam's charge per unit length λ'' is larger than its rest-frame value λ' by a factor $\gamma'' = 1/\sqrt{1 - (v''_p)^2/c^2}$:

$$\lambda'' = \frac{\lambda'}{\sqrt{1 - \frac{(v''_p)^2}{c^2}}} = \frac{1.972 \times 10^{-12}}{\sqrt{1 - \left(\frac{5.294 \times 10^7}{3.00 \times 10^8}\right)^2}}$$

$$= 2.003 \times 10^{-12} \text{ C/m}$$

Hence the electric field is directed away from the beam, and at a distance $r'' = r$, it has magnitude

$$\mathcal{E}'' = \frac{2\lambda''}{4\pi\epsilon_o r''} = \frac{2(8.99 \times 10^9)(2.003 \times 10^{-12})}{(1.00 \times 10^{-2})}$$

$$= 3.602 \text{ N/C}$$

Thus the electric force on the proton is $\vec{F}''_e = e\vec{\mathcal{E}}'' = (1.60 \times 10^{-19}) \times (3.602) = 5.76 \times 10^{-19}$ N away from the beam. The magnetic field lines are circles directed opposite to those found in part (a) and the field magnitude $\mathcal{B}'' = \mu_o i''/2\pi r''$. The current magnitude is

$$i'' = \lambda''\left|v''_p\right| = (2.003 \times 10^{-12})(5.294 \times 10^7)$$

$$= 1.060 \times 10^{-4} \text{ A}$$

so the magnetic field magnitude at distance r" is

$$\mathcal{B}'' = \frac{\mu_o i''}{2\pi r''} = \frac{\mu_o i''}{2\pi r} = \frac{(2 \times 10^{-7})(1.060 \times 10^{-4})}{(1.00 \times 10^{-2})}$$

$$= 2.120 \times 10^{-9} \text{ T}$$

Since $\vec{v}_p''$ is perpendicular to $\vec{\mathcal{B}}''$, the magnetic force has magnitude

$$F_m'' = e\left|v_p''\right|\mathcal{B}'' = (1.60 \times 10^{-19})(5.294 \times 10^7)(2.120 \times 10^{-9})$$
$$= \underline{1.80 \times 10^{-20}\ \text{N}}$$

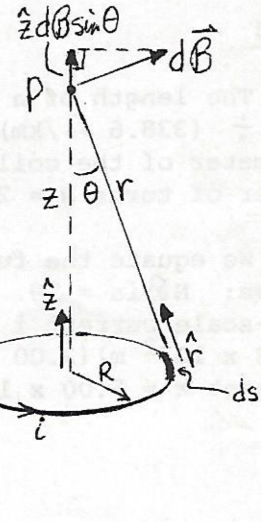
Since the beam and the proton are heading in the same direction, the magnetic force on the proton is directed toward the beam.

24-32

(a) If the only change in the experiment were the replacement of the "detector electron" by a "detector positron" with velocity equal to the drift velocity of the conduction electrons, the result would be the reversal of the direction of the Lorentz force on the "detector particle" due to the change in the sign of the charge.

(b) If the experimenter merely knew that the "detector particle" was moving at a velocity (speed and direction) identical with the drift velocity of the charge carriers, then only the following conclusions could be drawn from the experiment. If the "detector particle" were attracted by the wire, then the "detector particle" and the charge carriers must have charges of the same sign. If the "detector particle" were repelled by the wire, then the "detector particle" and the charge carriers must have charges of opposite sign. Specifically, if "detector particle" is identical to the current-carrying particles, then it will be attracted when $\vec{v} = \vec{v}_{drift}$ whether the carriers are matter or antimatter. Unless the experimenter had some independent way of determining the sign of the charges involved, the viewing of such experiments, as conducted in a distant galaxy, could not determine whether the galaxy is made of matter or antimatter. Specifically, an experiment involving a positron near a current-carrying anti-copper wire would not be distinguishable from an experiment involving an electron near a current-carrying copper wire.

24-33



The desired result is actually presented in Eqs. (23-45)-(23-47), but we also give a direct derivation here. A loop of radius R is shown in the figure at the right. The field contribution at P due to the current element $i\vec{ds}$ is given by the Biot-Savart law [Eq. (24-1)] as

$$d\vec{\mathcal{B}} = \frac{\mu_0}{4\pi}\frac{i}{r^2}\vec{ds} \times \hat{r}$$

When $d\vec{\mathcal{B}}$ is integrated only the z component survives, so that

$$\vec{\mathcal{B}} = \hat{z}\int (d\vec{\mathcal{B}}\cdot\hat{z}) = \hat{z}\int d\mathcal{B}\cdot\sin\theta$$

$$= \hat{z}\,\frac{\mu_o i}{4\pi}\int\frac{\sin\theta ds}{r^2}$$

But r and $\sin\theta$ are constant around the loop, with $r = \sqrt{R^2 + z^2}$ and $\sin\theta = R/r$. Since $\int ds = 2\pi R$, we find

$$\vec{\mathcal{B}} = \hat{z}\,\frac{\mu_o i}{4\pi}\,\frac{\sin\theta}{r^2}\int ds = \hat{z}\,\frac{\mu_o i}{4\pi}\left(\frac{R}{r^3}\right)(2\pi R)$$

$$= \frac{\mu_o}{4\pi}\,\frac{(2\pi R^2 i\hat{z})}{r^3} = \frac{\mu_o}{4\pi}\,\frac{2\vec{m}}{r^3}$$

since $\vec{m} \equiv i\pi R^2\hat{z}$. For $z \gg R$ the field on the z axis is

$$\vec{\mathcal{B}} = \left(\frac{\mu_o}{4\pi}\right)\cdot\frac{2m\hat{z}}{(z^2 + R^2)^{3/2}}$$

$$= \frac{\mu_o}{4\pi}\,\frac{2m\hat{z}}{z^3\left[1 + \left(\frac{R}{z}\right)^2\right]^{3/2}}$$

$$\simeq \frac{\mu_o}{4\pi}\,\frac{2\vec{m}}{z^3} \qquad (1)$$

as desired. According to Eqs. (21-28), the on-axis field far from an electric dipole is

$$\vec{\mathcal{E}} = \mathcal{E}_z\hat{z} = \frac{p\hat{z}}{4\pi\epsilon_o}\,\frac{(2z^2 - 0)}{(0 + z^2)^{5/2}} = \frac{2\vec{p}}{4\pi\epsilon_o z^3} \qquad (2)$$

The analogy between expressions (1) and (2) is evident.

24-34

(a) The length of a 1.00 - ohm piece of no. 30 copper wire is $1\,\Omega \div (338.6\,\Omega/\text{km}) = (10^5/338.6)$ cm = 295.3 cm. Since the perimeter of the coil is 2(2.50 + 2.00) = 9.00 cm, the required number of turns N = 295.3/9.00 = 32.8 $\simeq$ 33 turns.

(b) We equate the full-scale torques due to magnetic and spring forces: $N\mathcal{B}ia = k\theta$. Since the magnetic field $\mathcal{B}$ = 0.40 T, the full-scale current i = 1.00×10^{-3} A, the coil area a = $(2.50 \times 10^{-2}\text{ m})(2.00 \times 10^{-2}\text{ m}) = 5.00 \times 10^{-4}\text{ m}^2$, and the spring constant k = 5.00×10^{-6} N·m/rad, we find

$$\theta = \frac{N\mathcal{B}ia}{k} = \frac{(33)(0.40)(1.00 \times 10^{-3})(5.00 \times 10^{-4})}{(5.00 \times 10^{-6})}$$

$$= 1.32 \text{ rad} = 75.6° = \underline{76°}$$

24-35

(a) Referring to Fig. 24E-35, we consider first the circuit used for readings to 1 V. Ohm's law implies that $1\text{ V} = 10^{-3}\text{ A}(R_E + R)$. Since $R = 100\ \Omega$, we find

$$R_E = (1\text{ V}/10^{-3}\text{ A}) - 100\ \Omega = \underline{900\ \Omega} \qquad (1)$$

Examining the 10-V circuit, we obtain

$$10\text{ V} = 10^{-3}\text{ A}(R_E + R_F + R)$$

so that

$$R_F = (10\text{ V}/10^{-3}\text{ A}) - 1000\ \Omega = \underline{9000\ \Omega} \qquad (2)$$

Examining the 100-V circuit, we find

$$100\text{ V} = 10^{-3}\text{ A}(R_E + R_F + R_G + R)$$

so that

$$R_G = (100\text{ V}/10^{-3}\text{ A}) - 10{,}000\ \Omega = \underline{90{,}000\ \Omega} \qquad (3)$$

(b) Using the 0.1-A terminals, when the galvanometer current is $i_g = 10^{-3}$ A, the shunt current $i_s = 0.1\text{ A} - i_g = 0.099$ A. Since the galvanometer and the shunt are in parallel, we have

$$i_g R = i_s R_s \qquad (4)$$

But $R_s = R_B + R_C + R_D$ in this case, so that

$$(0.001\text{ A})(100\ \Omega) = (0.099\text{ A})(R_B + R_C + R_D)$$

or

$$99(R_B + R_C + R_D) = 100\ \Omega \qquad (5)$$

when the 1-A terminals are used, the shunt current at full scale is $i_s' = 1 - 0.001 = 0.999$ A. In this case, the shunt consists of resistors C and D, while resistor B is in series with the galvanometer. Hence the analogue of eq. (4) is

$$(0.001\ \text{A})(100\ \Omega + R_B) = (0.999\ \text{A})(R_C + R_D) \quad (6)$$

so that

$$999(R_C + R_D) = R_B + 100\ \Omega \quad (7)$$

When the 10-A terminals are used, the shunt current at full scale is $i_s'' = 10 - 0.001 = 9.999$ A. In this case, the shunt resistance $R_s'' = R_D$, while the path through the galvanometer has resistance $R'' = R + R_B + R_C$. The analogue of eq. (4) is therefore

$$(0.001\ \text{A})(100\ \Omega + R_B + R_C) = (9.999\ \text{A})(R_D) \quad (8)$$

so that

$$9999\ R_D = R_B + R_C + 100\ \Omega \quad (9)$$

Subtracting eq. (5) from eq. (7), we obtain

$$-99\ R_B + 900(R_C + R_D) = R_B$$

so that

$$R_B = 9(R_C + R_D) \quad (10)$$

Substituting eq. (10) into eq. (7), we find

$$111\ R_B = R_B + 100\ \Omega$$

so that

$$R_B = \frac{10}{11}\ \Omega \quad (11)$$

Then eq. (5) becomes

$$90\ \Omega + 99(R_C + R_D) = 100\ \Omega$$

so that

$$R_C + R_D = \frac{10}{99}\ \Omega \quad (12)$$

and eq. (9) becomes

$$9999\ R_D = \frac{10}{11}\ \Omega + R_C + 100\ \Omega$$

Using eq. (12) to eliminate R_D, this last equation is

$$9999\left(\frac{10}{99}\Omega - R_C\right) = \frac{10}{11}\Omega + R_C + 100\Omega \qquad (13)$$

Solving eq. (13) for R_C, we find

$$10{,}000\ R_C = 1010\ \Omega - 100\Omega - \frac{10}{11}\Omega$$

$$= \left(\frac{11{,}110 - 1{,}100 - 10}{11}\right)\Omega$$

$$= \left(\frac{10{,}000}{11}\right)\Omega$$

so that

$$R_C = \frac{1}{11}\Omega \qquad (14)$$

Then eq. (12) yields

$$R_D = \frac{10}{99}\Omega - R_C = \frac{10}{99} - \frac{9}{99} = \frac{1}{99}\Omega \qquad (15)$$

Summarizing the results of part (b), we have $R_B = \frac{10}{11}\Omega \simeq 0.9091\Omega$, $R_C = \frac{1}{11}\Omega \simeq 0.0909\ \Omega$, and $R_D = \frac{1}{99}\Omega \simeq 0.0101\ \Omega$.

<u>24-36</u>

(a) The surface charge density is $|q|/\pi r^2$. Hence the charge within a ring of radius R and width dR is

$$dq = \left(\frac{q}{\pi r^2}\right)\cdot(2\pi R dR) = \frac{2q}{r^2}\cdot R dR$$

The current carried by this ring is its charge divided by the rotation period:

$$di = \frac{dq}{(2\pi/\omega)} = \frac{q\omega}{\pi r^2} R dR$$

The magnetic moment contributed by this ring has magnitude dM $= a|di|$, where a is the area of the ring. Therefore

$$dM = \pi R^2 |di| = \frac{|q|\omega}{r^2} R^3 dR$$

Since the contributions $d\vec{M}$ from the various rings are all parallel, we have

$$M = \int dM = \int_{R=0}^{r} \frac{|q|\omega}{r^2} R^3 dR = \frac{|q|\omega}{r^3}\left(\left.\frac{R^4}{4}\right|_{R=0}^{R=r}\right)$$

....

$$= \frac{|q|\omega r^2}{4} \qquad (1)$$

(b) Assuming that the disk has a uniform mass distribution, we refer to Table 10-1 and find that its moment of inertia $I = \frac{1}{2}mR^2$. Hence its angular momentum has magnitude $L = \frac{1}{2}mR^2\omega$.

(c) The ratio is

$$\frac{M}{L} = \left(\frac{|q|\omega R^2}{4}\right) \frac{1}{(\frac{1}{2}mR^2\omega)} = \frac{|q|}{2m} \qquad (2)$$

If the charge q is positive, $\vec{M}$ and $\vec{L}$ are parallel. If the charge is negative, $\vec{M}$ and $\vec{L}$ are antiparallel. The relationship between $\vec{M}$ and $\vec{L}$ can be summarized by the equation

$$\vec{M} = \frac{q}{2m}\vec{L} \qquad (3)$$

[NOTE: Equation (3) can be shown to be valid for any macroscopic object (uniformly charged or not!), provided only that the ratio $\rho_q(\vec{r})/\rho_m(\vec{r})$ of the local charge density to the local mass density is constant throughout the object.]

24-37

(a) The magnitude ratio of the magnetic moment to angular momentum is

$$\frac{M_e}{L_e} = \frac{0.93 \times 10^{-23}\ \text{A}\cdot\text{m}^2}{0.53 \times 10^{-34}\ \text{kg}\cdot\text{m}^2/\text{s}} = 1.8 \times 10^{11}\ \text{C/kg}$$

while

$$\frac{e}{2m_e} = \frac{1.60 \times 10^{-19}\ \text{C}}{2(9.11 \times 10^{-31}\ \text{kg})} = 0.88 \times 10^{11}\ \text{C/kg}$$

Hence (M_e/L_e) is approximately twice the value that would be expected on the basis of Exercise 24-36.

(b) According to Table 10-1, the moment of inertia of a homogeneous sphere about any axis through its center is $I = \frac{2}{5}mr^2$. Hence the angular momentum of a spinning sphere is $L = I\omega = \frac{2}{5}mr^2\omega$. Since the speed of a point on the sphere's equator is $v = r\omega$, the angular momentum can be written as

$$L = \frac{2}{5}mrv$$

Solving this equation for v and using $L = 0.53 \times 10^{-34}\ kg\cdot m^2/s$, $m = 9.11 \times 10^{-31}$ kg, and $r = 2.8 \times 10^{-15}$ m, we find

$$v = \frac{5L}{2mr} = \frac{5(0.53 \times 10^{-34})}{2(9.11 \times 10^{-31})(2.8 \times 10^{-15})}$$

$$= 5.2 \times 10^{10}\ m/s$$

which is nearly 200 times the speed of light. The model of an electron as a tiny homogeneous spinning sphere is not tenable because it requires (impossible) "superluminal" speeds.

24-38

(a) The magnetic field in the core of the toroid is given by

$$\mathcal{B}_{int} = K_m \mu_o \frac{N}{\ell} i$$

$$= \frac{(1000)(4\pi \times 10^{-7}\ T\cdot m/A)(20)(10\ A)}{(0.25\ m)}$$

$$= 1.01\ T \qquad (1)$$

(b) In the presence of the air gap, we have

$$\mathcal{B}_{int}\left(\ell_1 + \frac{\ell_2}{K_m}\right) = \mu_o N i \qquad (2)$$

Solving eq. (2) for $\mathcal{B}_{int}$ and using $\ell_1 = 0.01$ m, $\ell_2 = \ell - \ell_1 = 0.24$ m, N = 20, i = 10 A, and $K_m = 1000$, we find

$$\mathcal{B}_{int} = \frac{\mu_o N i}{\left(\ell_1 + \frac{\ell_2}{K_m}\right)}$$

$$= \frac{(4\pi \times 10^{-7})(20)(10)}{\left(0.01 + \frac{0.24}{1000}\right)}$$

$$= \underline{2.45 \times 10^{-2}\ T} \qquad (3)$$

which is less than 3% of the original value given in eq. (1).

24-39

(a) According to Eq. (23-47a), the on-axis far field of a magnetic dipole is given by

$$\vec{\mathcal{B}} = \frac{\mu_o}{2\pi}\frac{\vec{m}}{r^3} \qquad (1)$$

If a second magnetic dipole carrying a dipole moment of the same magnitude is present in this field, the energy of orientation is $U = -\vec{m}' \cdot \vec{\mathcal{B}}$, by analogy with the equation $U = -\vec{p} \cdot \vec{\mathcal{E}}$ for the case of an electric dipole. (The expression $U = -\vec{m}' \cdot \vec{\mathcal{B}}$ can also be established directly from the torque equation $\vec{T} = \vec{m}' \times \vec{\mathcal{B}}$.) If the second dipole is aligned with the field produced by the first dipole, so that $\vec{m}' = \vec{m}$, the energy of orientation is

$$U_a = -\vec{m} \cdot \vec{\mathcal{B}} = -\frac{\mu_o m^2}{2\pi r^3} \qquad (2)$$

If the second dipole is unaligned -- that is, if the moment $\vec{m}'$ is randomly oriented with respect to $\vec{\mathcal{B}}$, then the average or expected energy of orientation is $U_u = \langle -\vec{m}' \cdot \vec{\mathcal{B}} \rangle$. This average equals $\langle -\vec{m}' \rangle \cdot \vec{\mathcal{B}}$, since the averaging is over the possible orientations of $\vec{m}'$, for which $\vec{\mathcal{B}}$ is a constant. But the average of any randomly oriented vector of given magnitude is $\vec{0}$. Hence we obtain

$$U_u = 0 \qquad (3)$$

Equations (2) and (3) imply that the average energy per dipole pair needed to destroy the alignment is

$$\Delta U = U_u - U_a = \frac{\mu_o m^2}{2\pi r^3} \qquad (4)$$

With $m = 2 \times 10^{-23}$ $A \cdot m^2$ and $r = 1 \times 10^{-10}$ m, eq. (4) yields

$$U = \frac{(2 \times 10^{-7})(2 \times 10^{-23})^2}{(1 \times 10^{-10})^3}$$

$$= \underline{8 \times 10^{-23} \text{ J} = 5 \times 10^{-4} \text{ eV}}$$

(b) At temperature T_c, the thermal kinetic energy associated with rotational degrees of freedom is of order kT_c, where k is Boltzmann's constant. With $k = 1.38 \times 10^{-23}$ J/K and $T_c = 1043$ K, kT_c = $\underline{1.4 \times 10^{-20} \text{ J} = 9 \times 10^{-2} \text{ eV}}$, which is about 200 times as large as the energy required to overcome the strictly magnetic torques.

CHAPTER TWENTY-FIVE

25-1

The entire circuit is moving (without changing its shape) through a uniform magnetic field, so the magnetic flux through the circuit remains constant. Hence the emf induced in the circuit is zero.